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«КИЇВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ
імені ІГОРЯ СІКОРСЬКОГО»

МЕХАНІКО-МАШИНОБУДІВНИЙ ІНСТИТУТ

КАФЕДРА ДИНАМІКИ І МІЦНОСТІ МАШИН ТА ОПОРУ МАТЕРІАЛІВ

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______ Сергій ПИСКУНОВ
«___»____________2020 р.

Магістерська дисертація

на здобуття ступеня магістра

за освітньо-професійною програмою «Динаміка і міцність машин»

зі спеціальності 131 «Прикладна механіка»

на тему: «Обґрунтування вибору конструктивного виконання вуглепластикової панелі крила з умов міцності, жорсткості та стійкості»

Виконав (-ла):
студент (-ка) VI курсу, групи МП-91мп
Динник Євген Вячеславович ___________

Керівник:
д. т. н., професор
Сидоренко Юрій Михайлович ___________

Рецензент:
д.т.н., проф.
Данильченко Ю.М. ___________

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Студент (-ка) ___________

Київ – 2020 року

Національний технічний університет України

«Київський політехнічний інститут імені Ігоря Сікорського»

Механіко-машинобудівний інститут

Кафедра динаміки і міцності машин та опору матеріалів

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Спеціальність – 131 «Прикладна механіка»

Освітньо-професійна програма – «Динаміка і міцність машин»

ЗАТВЕРДЖУЮ

Завідувач кафедри

________ Сергій ПИСКУНОВ

«___»_____________2020 р.

ЗАВДАННЯ
на магістерську дисертацію студенту

Динник Євген Вячеславович

1. Тема дисертації «Обґрунтування вибору конструктивного виконання вуглепластикової панелі крила з умов міцності, жорсткості та стійкості», науковий керівник дисертації Сидоренко Юрій Михайлович, д. т. н., професор затверджені наказом по університету від «___»__________2020 р. № ____

2. Термін подання студентом дисертації: 10.12.2020 р.

3. Об’єкт дослідження: композитні вуглепластикові панелі крила.

4. Предмет дослідження: вибір оптимального виконання вуглепластикової панелі.

5. Перелік завдань, які потрібно розробити:

1) Огляд варіантів виконання кесону крила з вуглепластику.
2) Дослідження та порівняння пружних характеристик вуглестрічка та вуглеткинини для різних кутів навантаження.
3) Визначення жорсткістних характеристик та критичних зусиль втрати стійкості композитних панелей та вибір найбільш оптимального виконання.

4) Розрахунок на міцність вуглепластикової панелі.
5) Розробка стартап–проекту.

6. Орієнтовний перелік графічного (ілюстративного) матеріалу – 6 і більше.

7. Орієнтовний перелік публікацій – 1 і більше.

8. Дата видачі завдання: 01.09.2020 р.

Календарний план

№ з/п	Назва етапів виконання магістерської дисертації	Термін виконання етапів магістерської дисертації	Примітка
1	Аналіз літератури	01.09.20 - 19.09.20	
2	Розрахунок пружних характеристик для вуглестрічка та ввуглетканини.	20.09.20 – 09.10.20	
3	Розрахунок жорсткістних характеристик панелей	10.10.20 – 31.10.20	
4	Визначення критичних зусиль та розрахунок на міцність	01.11.20 – 19.11.20	
5	Стартап-проект та висновки	20.11.20 – 10.12.20	

Студент ____________ Євген ДИННИК

Науковий керівник ____________ Юрій СИДОРЕНКО

ABSTRACT

The master`s degree dissertation for the amount of work is 79 pages, 42 figures, 33 tables, and contains 10 literature.

The object of the work is the set of carbon fiber composite panels of wing.

The main goal of this dissertation is the choose optimal configuration of carbon fiber composite panel of wing.

Relevance: Parts of composite materials have complex methods of analysis. At the same time, a large number of parts of modern aircraft structure make of composite materials and it is important to select optimal configuration of part, which made of composite materials. The relevance is to solve this problem using the analytical method in Microsoft Excel.

The carbon fiber panels are analyzed using the Microsoft Excel.

As a result of this work, it was calculated stiffness properties and critical load for a large number of panels and compared this results. Also was performed strength analysis of composite panel.

РЕФЕРАТ

Дана магістерська дисертація за обсягом роботи складає 79 сторінок, 42 ілюстрації, 33 таблиці та містить 10 літературних джерел.

Об'єктом дослідження є набор вуглепластикових панелей крила.

Головна ціль даної дисертації – вибір оптимальної конфігурації вуглепластикової панелі крила.

Актуальність: Деталі з композиційних матеріалів мають складні методи аналізу. Разом з тим, велика кількість деталей конструкції сучасного літака виконано з композиційних матеріалів, тому важливо вибрати оптимальну конфігурацію деталі, виготовленої з композиційних матеріалів. Актуальним є вирішення цієї проблеми за допомогою аналітичного методу в Microsoft Excel.

Панелі з вуглецевого волокна аналізуються за допомогою Microsoft Excel.

В результаті цієї роботи було розраховано властивості жорсткості та критичне навантаження для великої кількості панелей та порівняно ці результати. Також був проведений аналіз міцності композитної панелі.

Contents

INTRODUCTION

Until recently, most of the typical aircraft design consist from aluminum alloys. Aluminum alloys have quite high specific strength, stiffness and high manufacturability. But the desire to reduce the mass of the structure and at the same time remain or even increase the strength and stiffness led to creating composite materials. And last time many companies which produce commercial airplanes started use large quantity of composite materials in the airframe. The early days of composite materials, they were used secondary structure, which does not carry significant loads and these failure will not led to catastrophic effects. However, airplanes of last generation of commercial have primary parts produced from composite materials, such as fuselage skins, wing panels and spars, empennage panels and spars and other structures. Outstanding representatives are such airplanes like Boeing 787 Dreamliner, 777X, Airbus A350 XWB, A220.

Composite material it is artificially created inhomogeneous continuous material, which consist of two or more components whit clear dividing boundary between them. The most composites (except laminated composite) consist from matrix and reinforce elements, which included into the matrix. Mechanical properties of composite material depends from relationship between properties of reinforce elements and matrix, and from strength of the connection between them.

As a result of the combination of the reinforcing elements and the matrix, a complex of new properties is formed, which gives not only initial properties of components, but also received material includes new properties, which components of composite are not have. In particular, the presence of interfaces between the reinforcing elements and the matrix significantly increases the crack resistance of the material. Also, for composites, unlike homogeneous materials, increasing of static strength leads to the increasing of impact strength.

But despite all the advantages of composite materials, they have a number of disadvantages. Structural analysis of composites is more difficult than analysis for metals. Behavior of composite under cyclic and impact loads poorly studied, so for aircraft composite structures principally loaded by cyclic load, used high safety factors.

Therefore, it so important to study the behavior of composites under different loads, understanding analysis approaches for composites and improving existing approaches for future developments.

Carbon fiber reinforced polymer (CFRP) or carbon composite is an extremely strong and light fiber-reinforced plastic which contains carbon fibers. CFRPs can be expensive to produce, but commonly used wherever high strength-to-weight ratio and stiffness are required, such as aerospace, superstructures of ship, automotive, civil engineering, sports equipment, and an increasing number of consumer and technical applications. The binding polymer is often a thermoset resin such as epoxy.

CFRP consists from many plies (laminas), which have unidirectional or cross-directed fibers jointed by matrix. Layers in turn jointed by resin.

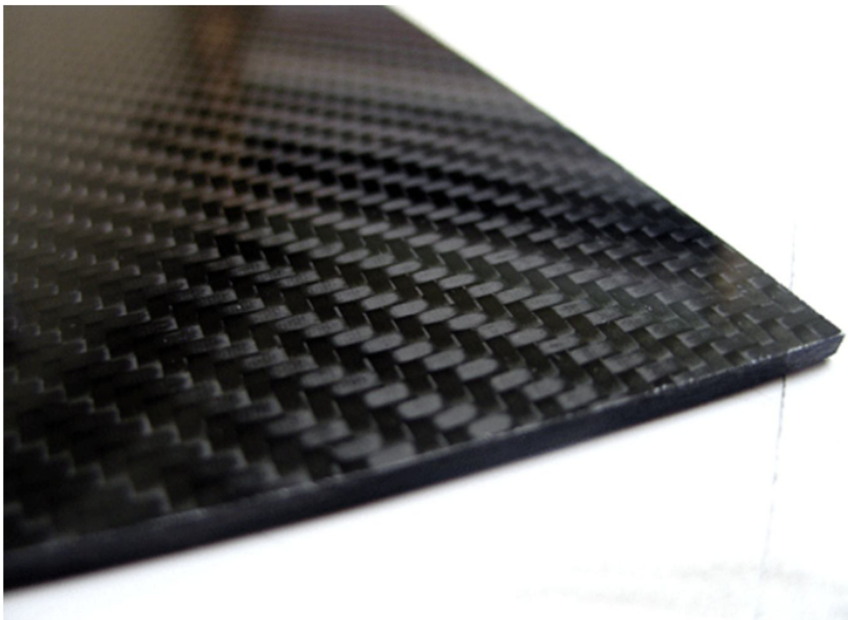


Fig. 1 Carbon fiber reinforced polymer

1. EXISTING TYPES OF CARBON FIBER PANELS OF WING

There are three main types of wing box made of composite materials: composite multi-rib structural layout with reinforced panels, composite multi-spar structural layout with monolithic panels without reinforcement and sandwich skin panels structural layout. In practice, the listed types of wingbox are often combined for getting better properties.

1.1 REINFORCED COMPOSITE PANELS

Reinforced composite panels are as similar as possible like classical metallic panels with stringers. The only difference is the material. Longitudinal orientated stringers and transvers orientated ribs act as reinforcing elements for buckling.

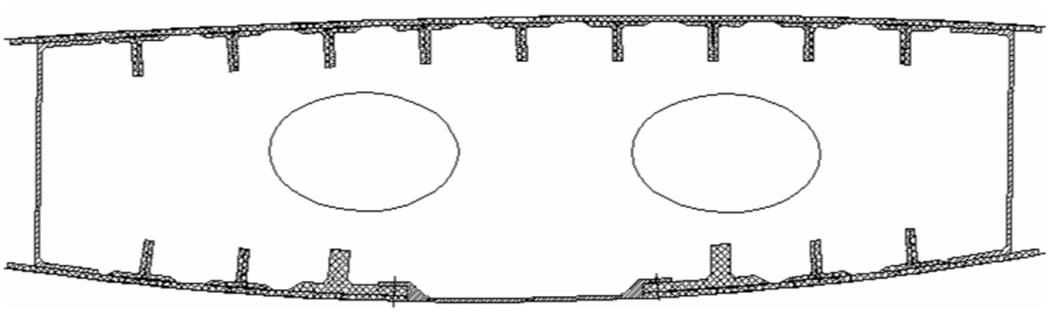


Fig.2 Composite multi-rib structural layout with reinforced panels

Reinforced panels are most common type of wing panels. Almost all commercial airplanes have outer wing with panels reinforced by stringers. Obviously, composite reinforced panels have the same advantages like for metallic reinforced panels, but they also have their own advantages and disadvantages.

The box formed by reinforced panels and two spars as a whole resist bending, where panels primarily resist axial loads and shear loads due torque, while webs of spars resist shear loads due bending and torque. Reinforced elements resist local buckling between ribs and these elements and add moment of inertia to resist to resist global buckling of entire panel.

Table 1 – Advantages and disadvantages of multi-rib structural layout with reinforced panels

Advantages	Disadvantages
- The possibility to produce the panels without the autoclave process and RTM; - It can be manufactured in one production cycle, if the stringer has a T-shaped cross-section; - A traditional structural layout, tried-and-true on metal wings; - There is a possibility to splice different systems (hydraulic, fuel, etc.) to the stringers; - There are composite analogues, and the level of technical risk is the lowest;	- The complexity of sealing ribs; - Complex configuration of the panels; - The complexity of manufacturing the panels in one production cycle, if the stringer has a I-shaped or more complex cross-section.

1.2 MULTI-SPAR COMPOSITE WINGBOX WITH MONOLITHIC PANELS

Multi-spar composite structure with panels without reinforced less common than reinforced panels, but in the some specific parts of wing multi-spar structure more preferable. For modern aircrafts, multi-spar structure often utilizes for wing center section and winglets.

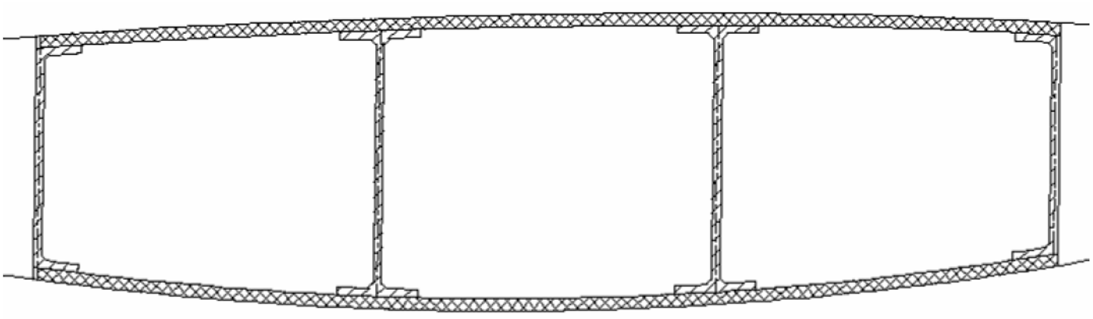


Fig.3 Composite multi-spar structural layout with monolithic panels without reinforcement

Fig. 4 Multi-spar winglet structure

The multi-spar structure works similar to reinforced panels with two spars, but additional spars adds torsional stiffenes for getting box with multiple closed contours and spars reinforce panels instead stringers in the reinforced panels.

Table 2 - Advantages and disadvantages of multi-spar structural layout with monolithic panels

Advantages	Disadvantages
- Ease of manufacturing the panels; - Simple design and ease of ribs` sealing; - Increased stiffness of the outer wing panel, which is especially important when using composite materials for the designed structure; - Rational technological spacing of manholes at the bottom panel due to the small number of ribs; - The opportunity to move to an integrated wingbox design and "no-center-section" wing with the joint of the outer wing panel at the surface of the aircraft symmetry;	- Larger specific weight of regular zones of the panels (compared to the stringer design) due to the greater thickness of the coverings necessary for their sustainability; - In the root zone of the rear spar there are areas difficult to reach for building the wingbox, installation and maintenance of the zone systems; - No analogues for long-range aircraft, high level of technical risk. - In order to prevent thick panels from buckling the walls must possess significant stiffness

1.3 SANDWICH SKIN PANELS

A sandwich panel is any structure made of three layers: a low-density core, and a thin skin-layer bonded to each side. The skin-layers can be either metal or fiber-epoxy material. The structural functionality of a sandwich panel is similar the classic I-beam, where two face primarily resist the in-plane and lateral bending bending loads (similar to flanges of an I-beam), while the core material mainly resists the shear loads.

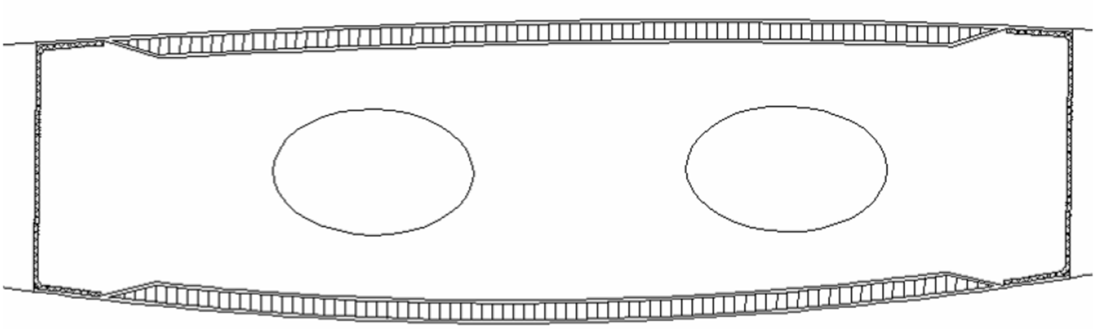


Fig. 5 Sandwich skin panels structural layout

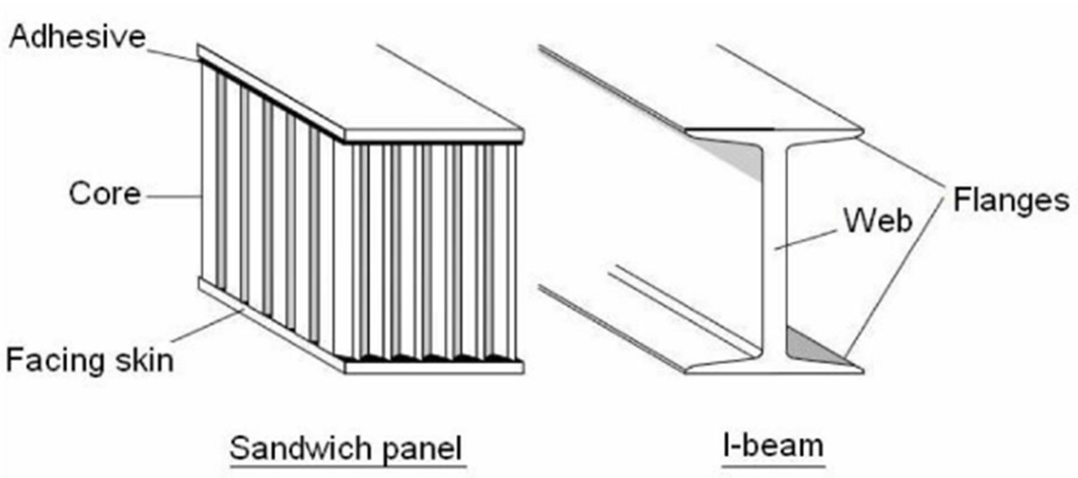


Fig. 6 Sandwich panel and I-beam

Wingbox assembled from sandwich panels and two spars works similar to wingbox with reinforced panels and multi-spar wingbox. Since facing skins separated by some distance, moment of inertia of panel increased, that it critical buckling loads also increased.

Table 3 - Advantages and disadvantages of multi-spar structural layout with monolithic panels

Advantages	Disadvantages
- It is possible to reduce the weight of the wing panels in comparison with other structural layout; - Possibility of manufacturing the panels with a honeycomb filler in one production cycle using combined technologies; - Ease of manufacturing the panels.	- It is difficult to determine the place of unsticking of the covering from honeycomb filler as a result of hitting or in the process of exploitation; - Potential problems with accumulation of moisture in the honeycomb filler when the airtight covering is damaged; - Inability to produce panels with honeycomb filler through RTM technology; - The necessity to use autoclaves in manufacturing the panels.

1.4 OVERVIEW OF MANUFACTURING FOR COMPOSITE MATERIALS

There are three main methods of manufacturing of laminates: lay-up, winding and pultrusion.

1.4.1 Lay-up

Resins are impregnated by hand into fibres which are in the form of woven, knitted, stitched or bonded fabrics. This is usually accomplished by rollers or brushes, with an increasing use of nip-roller type impregnators for forcing resin into the fabrics by means of rotating rollers and a bath of resin. Laminates are left to cure under standard atmospheric conditions.

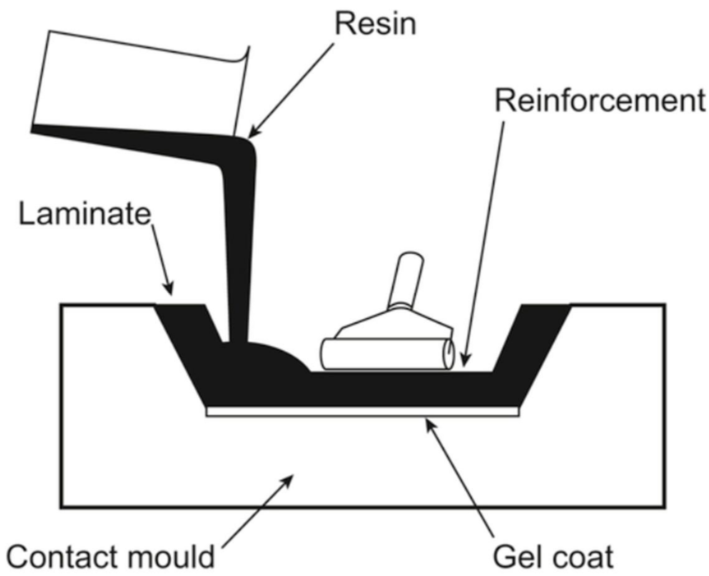


Fig. 7 Lay-up

1.4.2 Winding

Winding is a simple and effective method for producing parts such as pipes and cylindrical containers in a wide range of sizes using both prepreg and other methods. Part diameters ranging from 25mm to 6m are commonly fabricated. The process consists of wrapping bands of continuous fiber or roving over a mandrel in a controlled operation. The production cycle for most filament wound composites can be subdivided into the following stages.

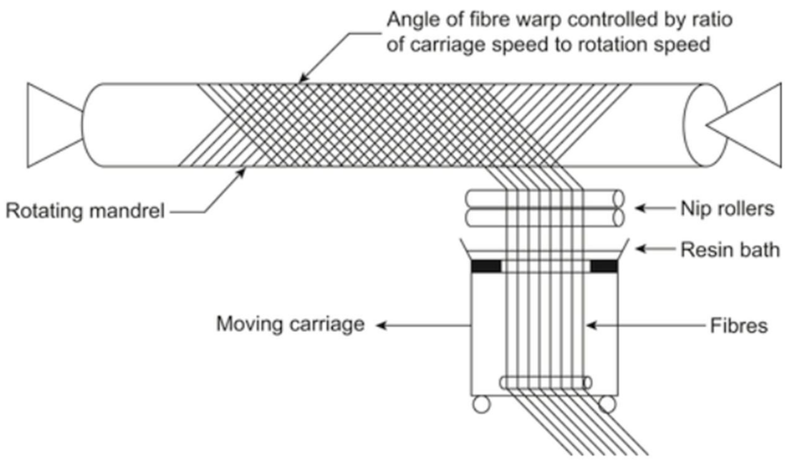


Fig. 8 Winding

1.4.3 Pultrusion

Pultrusion is a cost-effective automated process for manufacturing continuous, constant cross-section composite profiles. It is similar to the extrusion process used for manufacturing metallic aircraft stringers except the material is pulled through a die instead of being pushed through. Raw materials are a liquid resin mixture (containing resin, fillers and specialized additives) and flexible textile reinforcing fibers.

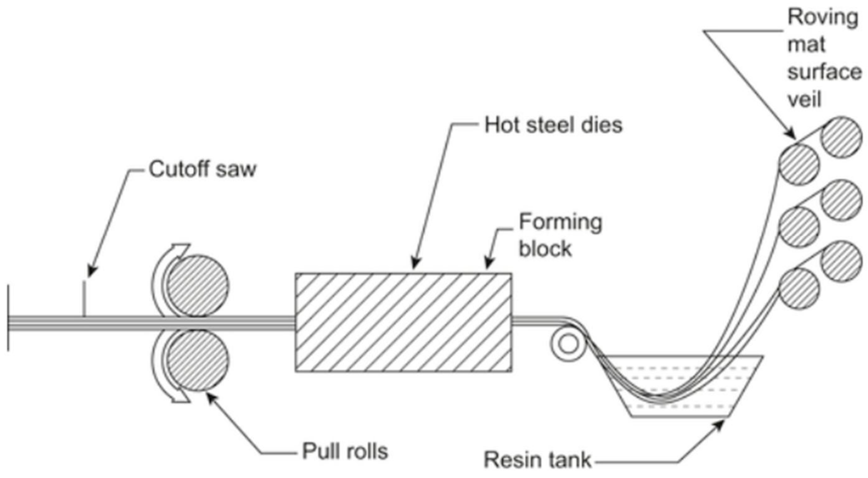


Fig. 9 Pultrusion

1.5 TYPES OF LAMINAS

There two main types of laminas: tape and fabric. Tape is a one ply of unidirectional fibers. Tape has high elastic and strength properties in the direction of the fibers and very low properties in the transverse direction. Fabric consists of intertwined by angle of 90° fibers. Fabric has quiet high properties in the longitude and transverse directions.

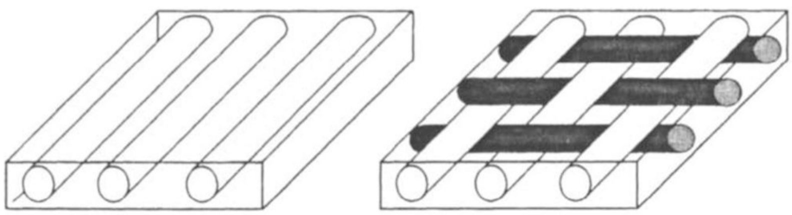


Fig. 10 Tape (right) and fabric (left)

2. STIFFNESS MATRICES OF THIN LAMINATES

Continuous fiber-reinforced composite materials often have easily identifiable preferred directions associated with fiber orientation or symmetry planes. Therefore local and global Cartesian coordinate systems are used for analysis of composite materials. Local Cartesian coordinate system x_1, x_2, x_3 have axis's aligned with fibers or symmetry planes. Global Cartesian coordinate system x, y, z attached to a fixed reference point.

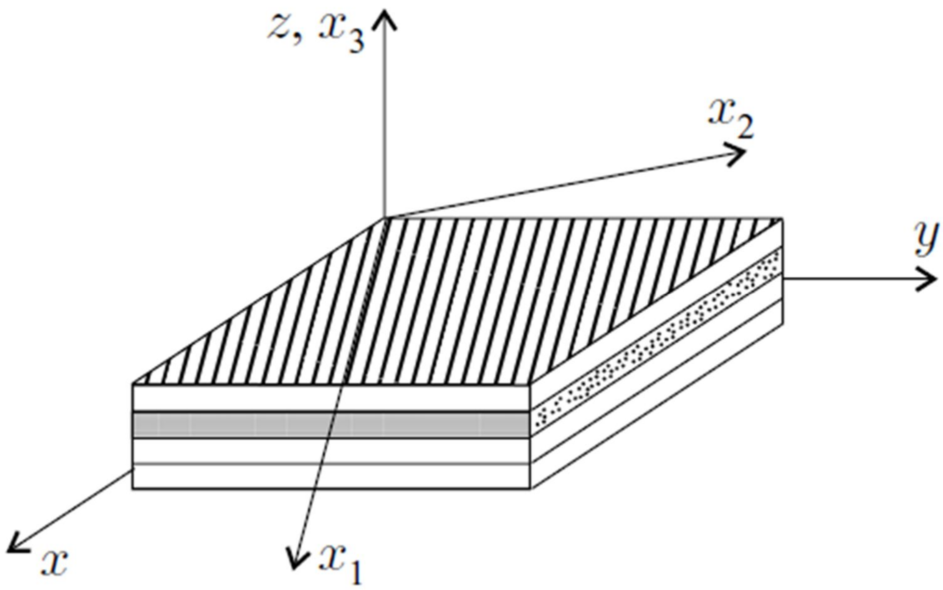


Fig.11 The global x, y, z and local x_1, x_2, x_3 coordinate systems [2]

In general in the x, y, z global coordinate system, the stress-strain relationship [2] are

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{C}_{11} & \overline{C}_{12} & \overline{C}_{13} & \overline{C}_{14} & \overline{C}_{15} & \overline{C}_{16} \\ \overline{C}_{21} & \overline{C}_{22} & \overline{C}_{23} & \overline{C}_{24} & \overline{C}_{25} & \overline{C}_{26} \\ \overline{C}_{31} & \overline{C}_{32} & \overline{C}_{33} & \overline{C}_{34} & \overline{C}_{35} & \overline{C}_{36} \\ \overline{C}_{41} & \overline{C}_{42} & \overline{C}_{43} & \overline{C}_{44} & \overline{C}_{45} & \overline{C}_{46} \\ \overline{C}_{51} & \overline{C}_{52} & \overline{C}_{53} & \overline{C}_{54} & \overline{C}_{55} & \overline{C}_{56} \\ \overline{C}_{61} & \overline{C}_{62} & \overline{C}_{63} & \overline{C}_{64} & \overline{C}_{65} & \overline{C}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{2.1}$$

where $\overline{C}_{ij}$ are the elements of the stiffness matrix $[\overline{C}]$ in the x, y, z coordinate system.

Inversion of Eq.(2.1) results in the following strain-stress relationships:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} & \overline{S}_{13} & \overline{S}_{14} & \overline{S}_{15} & \overline{S}_{16} \\ \overline{S}_{21} & \overline{S}_{22} & \overline{S}_{23} & \overline{S}_{24} & \overline{S}_{25} & \overline{S}_{26} \\ \overline{S}_{31} & \overline{S}_{32} & \overline{S}_{33} & \overline{S}_{34} & \overline{S}_{35} & \overline{S}_{36} \\ \overline{S}_{41} & \overline{S}_{42} & \overline{S}_{43} & \overline{S}_{44} & \overline{S}_{45} & \overline{S}_{46} \\ \overline{S}_{51} & \overline{S}_{52} & \overline{S}_{53} & \overline{S}_{54} & \overline{S}_{55} & \overline{S}_{56} \\ \overline{S}_{61} & \overline{S}_{62} & \overline{S}_{63} & \overline{S}_{64} & \overline{S}_{65} & \overline{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} \tag{2.2}$$

Where $\overline{S}_{ij}$ are the elements of compliance matrix $[\overline{S}]$ in the x, y, z coordinate system.

In the x_1, x_2, x_3 coordinate system the stress-strain relationship [2] are

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} \tag{2.3}$$

where C_{ij} are the elements of the stiffness matrix $[C]$ in the x_1, x_2, x_3 coordinate system.

By inverting Eq.(2.3) the following strain-stress relationships are obtained:

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} \tag{2.4}$$

Where S_{ij} are the elements of compliance matrix $[S]$ in the x_1, x_2, x_3 coordinate system.

It is evident from Eqs. (1)-(4) that the compliance matrix $[S]$ is the inverse of the stiffness matrix $[C]$:

$$[\overline{S}] = [\overline{C}]^{-1} \qquad [S] = [C]^{-1} \tag{2.5}$$

The compliance matrix for isotropic material [2]:

$$[S] = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix} \tag{2.6}$$

The compliance matrix for orthotropic material [2]:

$$[S] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \tag{2.7}$$

And stiffness matrix for isotropic and orthotropic materials looks following [2]:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \tag{2.8}$$

The Eq.(2.8) can be written in the following form [2]:

$$[C] = \begin{bmatrix} [L] & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & [M] \end{bmatrix} \tag{2.9}$$

Where $[L]$ and $[M]$ submatrices with following elements [2]:

Orthotropic

$$[L] = \frac{1}{D} \begin{bmatrix} E_1\left(1 - \frac{E_3}{E_2}\nu_{23}^2\right) & E_2\left(\nu_{12} + \frac{E_3}{E_2}\nu_{13}\nu_{23}\right) & E_3\left(\nu_{13} + \nu_{12}\nu_{23}\right) \\ E_2\left(\nu_{12} + \frac{E_3}{E_2}\nu_{13}\nu_{23}\right) & E_2\left(1 - \frac{E_3}{E_1}\nu_{13}^2\right) & E_3\left(\nu_{23} + \frac{E_2}{E_1}\nu_{12}\nu_{13}\right) \\ E_3\left(\nu_{13} + \nu_{12}\nu_{23}\right) & E_3\left(\nu_{23} + \frac{E_2}{E_1}\nu_{12}\nu_{13}\right) & E_3\left(1 - \frac{E_2}{E_1}\nu_{12}^2\right) \end{bmatrix} \tag{2.10}$$

$$D = \frac{E_1E_2E_3 - \nu_{23}^2E_1E_3^2 - \nu_{12}^2E_2^2E_3^2 - 2\nu_{12}\nu_{13}\nu_{23}E_2E_3^2 - \nu_{13}^2E_2E_3^2}{E_1E_2E_3} \tag{2.11}$$

$$[M] = \begin{bmatrix} G_{23} & 0 & 0 \\ 0 & G_{13} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \tag{2.12}$$

Isotropic

$$[L] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \tag{2.13}$$

$$[M] = \begin{bmatrix} \frac{E}{2(1+\nu)} & 0 & 0 \\ 0 & \frac{E}{2(1+\nu)} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \tag{2.14}$$

The stiffness matrix for in-plane forces and moments has following view [2]:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.15}$$

The vectors on the left and right hand side represent generalized forces and strains.

2.1 ELASTIC PROPERTIES OF LAMINA

Each layer (lamina) of composite material consists from unidirectional fibers which defines direction of layer and from matrix which provides normal and transversal stiffness of lamina. Such lamina is orthotropic, since has two mutual planes of symmetry. The stress strain relationship for looks like follow [6]:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11}^0 & C_{12}^0 & 0 \\ C_{12}^0 & C_{22}^0 & 0 \\ 0 & 0 & C_{66}^0 \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{2.15}$$

where

σ_1, σ_2, τ_{12} – stresses which applied to lamina;

ε_1, ε_2, γ_{12} – deformation of lamina;

C_{ij}^0 - elements of stiffness matrix of lamina.

The elements of stiffness matrix of lamina calculates by following equations [6]:

$$C_{11}^{0} = \frac{E_1}{1 - \nu_{12} \cdot \nu_{21}} \tag{2.16}$$

$$C_{12}^{0} = \frac{E_1 \cdot \nu_{21}}{1 - \nu_{12} \cdot \nu_{21}} = \frac{E_2 \cdot \nu_{12}}{1 - \nu_{12} \cdot \nu_{21}} \tag{2.17}$$

$$C_{22}^{0} = \frac{E_2}{1 - \nu_{12} \cdot \nu_{21}} \tag{2.18}$$

$$C_{66}^{0} = G_{12} \tag{2.19}$$

where

E_1, E_2 – Young's moduli in the longitude and transvers directions of lamina;

G_{12} – shear modulus of lamina;

ν_{12} – major Poison ratio;

ν_{21} – minor Poison ratio which determines by Maxwell equation [6]:

$$\nu_{12} \cdot E_2 = \nu_{21} \cdot E_1 \tag{2.20}$$

If lamina loaded in direction, which doesn't coincide with axes of lamina than lamina is in state of layer-by-layer loading like part of laminate. And than stress-strain relationship takes the form [6]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11}^{\phi} & C_{12}^{\phi} & C_{16}^{\phi} \\ C_{13}^{\phi} & C_{22}^{\phi} & C_{26}^{\phi} \\ C_{16}^{\phi} & C_{26}^{\phi} & C_{66}^{\phi} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2.21}$$

where $\mathrm{C}_{\mathrm{ij}}^{\varphi}$ – elements of stiffness matrix of lamina which rotated by angle φ [6]:

$$C_{11}^{\varphi} = V_1 + V_2 \cdot cos2\varphi + V_3 \cdot cos4\varphi \tag{2.22}$$

$$C_{12}^{\varphi} = V_1 - 2 \cdot V_4 - V_3 \cdot cos4\varphi \tag{2.23}$$

$$C_{16}^{\varphi} = 0.5 \cdot V_2 \cdot sin2\varphi + V_3 \cdot sin4\varphi \tag{2.24}$$

$$C_{22}^{\varphi} = V_1 - V_2 \cdot cos2\varphi + V_3 \cdot cos4\varphi \quad (2.25)$$

$$C_{26}^{\varphi} = 0.5 \cdot V_2 \cdot sin2\varphi - V_3 \cdot sin4\varphi \quad (2.26)$$

$$C_{66}^{\varphi} = V_4 - V_3 \cdot cos4\varphi \quad (2.27)$$

Here independent coefficients V_1, V_2, V_3 and V_4 determines by following equations [6]:

$$V_1 = (3 \cdot C_{11}^0 + 2 \cdot C_{12}^0 + 3 \cdot C_{22}^0 + 4 \cdot C_{66}^0)/8 \quad (2.28)$$

$$V_2 = (C_{11}^0 - C_{22}^0)/2 \quad (2.29)$$

$$V_3 = (C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 - 4 \cdot C_{66}^0)/8 \quad (2.30)$$

$$V_4 = (C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 + 4 \cdot C_{66}^0)/8 \quad (2.31)$$

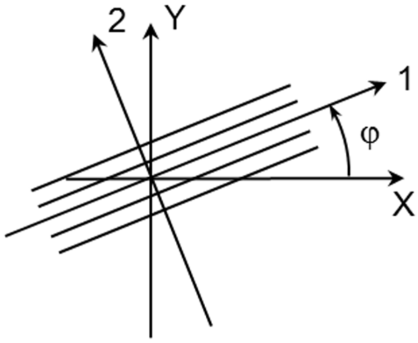


Fig. 12 Lamina rotated by angle φ relative to coordinate system of laminate

Coefficients V_1 and V_4 describe average tension and shear stiffness's, and coefficients V_2 and V_3 describe degree of anisotropy of material.

Thus, behavior of lamina in plane-stress condition describes by four independent elastic constants:

E_1, E_2, G_{12}, ν_{12} – for angels of reinforcing 0° and 90°;

V_1, V_2, V_3, V_4 – for angels of reinforcing φ°.

Elastic properties of lamina which rotated by angle φ° [6]:

$$E_x = \frac{\Delta C}{C_{22}^{\varphi} \cdot C_{66}^{\varphi} - \left(C_{26}^{\varphi}\right)^2} \tag{2.32}$$

$$E_y = \frac{\Delta C}{C_{11}^{\varphi} \cdot C_{66}^{\varphi} - \left(C_{16}^{\varphi}\right)^2} \tag{2.33}$$

$$G_{xy} = \frac{\Delta C}{C_{11}^{\varphi} \cdot C_{22}^{\varphi} - \left(C_{12}^{\varphi}\right)^2} \tag{2.34}$$

$$\nu_{xy} = \frac{C_{12}^{\varphi} \cdot C_{66}^{\varphi} - C_{16}^{\varphi} \cdot C_{26}^{\varphi}}{C_{22}^{\varphi} \cdot C_{66}^{\varphi} - \left(C_{26}^{\varphi}\right)^2} \tag{2.35}$$

where ΔC – determinant of stiffness matrix:

$$\Delta C = det \begin{bmatrix} C_{11}^{\phi} & C_{12}^{\phi} & C_{16}^{\phi} \\ C_{12}^{\phi} & C_{22}^{\phi} & C_{26}^{\phi} \\ C_{16}^{\phi} & C_{26}^{\phi} & C_{66}^{\phi} \end{bmatrix} \tag{2.36}$$

There are two types of plies: tape and fabric. Tapes and fabrics have own advantages. For example, tape has higher properties in the direction of the fibers, and fabric has high manufacturing properties. The table 2 provides elastic properties of tape and fabric.

Table 4 – Elastic properties of tape and fabric laminas

Material	E_1, MPa	E_2, MPa	G_{12}, MPa	ν_{12}	δ_m, mm
Tape	143000	8400	5600	0.36	0.125
Fabric	65000	63000	6500	0.07	0.125

Tape

Secondary Poisson ratio:

$$\nu_{21} = \nu_{12} \cdot \frac{E_2}{E_1} = 0.36 \cdot \frac{8400}{143000} = 0.021.$$

Coefficients of stiffness matrix of lamina:

$$C_{11}^0 = \frac{E_1}{1 - \nu_{12} \cdot \nu_{21}} = \frac{143000}{1 - 0.36 \cdot 0.021} = 144097\ MPa$$

$$C_{12}^0 = \frac{E_1 \cdot \nu_{21}}{1 - \nu_{12} \cdot \nu_{21}} = \frac{E_2 \cdot \nu_{12}}{1 - \nu_{12} \cdot \nu_{21}} = 3047\ MPa$$

$$C_{22}^0 = \frac{E_2}{1 - \nu_{12} \cdot \nu_{21}} = 8464\ MPa$$

$$C_{66}^0 = G_{12} = 5600\ MPa$$

Independent coefficients:

$$V_1 = \frac{3 \cdot C_{11}^0 + 2 \cdot C_{12}^0 + 3 \cdot C_{22}^0 + 4 \cdot C_{66}^0}{8} = \frac{3 \cdot 144097 + 2 \cdot 3047 + 3 \cdot 8464 + 4 \cdot 5600}{8} = 60772\ MPa$$

$$V_2 = \frac{C_{11}^0 - C_{22}^0}{2} = \frac{144097 - 8464}{2} = 67816\ MPa$$

$$V_3 = \frac{C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 - 4 \cdot C_{66}^0}{8} = \frac{144097 - 2 \cdot 3047 + 8464 - 4 \cdot 5600}{8} = 15508\ MPa$$

$$V_4 = \frac{C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 + 4 \cdot C_{66}^0}{8} = \frac{144097 - 2 \cdot 3047 + 8464 + 4 \cdot 5600}{8} = 21108\ MPa$$

Coefficients of stiffness matrix of lamina rotated on angle $\varphi = 45°$:

$$C_{11}^{45} = V_1 + V_2 \cdot cos2\varphi + V_3 \cdot cos4\varphi$$
$$= 60772 + 67816 \cdot \cos(90°) + 15508 \cdot cos(180°) = 45264\ MPa$$

$$C_{12}^{\varphi} = V_1 - 2 \cdot V_4 - V_3 \cdot cos4\varphi = 60772 - 2 \cdot 21108 - 15508 \cdot cos(180°)$$
$$= 34064\ MPa$$

$$C_{16}^{\varphi} = 0.5 \cdot V_2 \cdot sin2\varphi + V_3 \cdot sin4\varphi = 0.5 \cdot 67816 \cdot sin(90°) + 15508 \cdot sin(180°)$$
$$= 33908\ MPa$$

$$C_{22}^{\varphi} = V_1 - V_2 \cdot cos2\varphi + V_3 \cdot cos4\varphi$$
$$= 60772 - 67816 \cdot \cos(90°) + 15508 \cdot cos(180°) = 45264\ MPa$$

$$C_{26}^{\varphi} = 0.5 \cdot V_2 \cdot sin2\varphi - V_3 \cdot sin4\varphi = 0.5 \cdot 67816 \cdot sin(90°) - 15508 \cdot sin(180°)$$
$$= 33908\ MPa$$

$$C_{66}^{\varphi} = V_4 - V_3 \cdot cos4\varphi = 21108 - 15508 \cdot cos(180°) = 36617\ MPa$$

Determinant of stiffness matrix:

$$\Delta C = det\begin{bmatrix} C_{11}^{\phi} & C_{12}^{\phi} & C_{16}^{\phi} \\ C_{12}^{\phi} & C_{22}^{\phi} & C_{26}^{\phi} \\ C_{16}^{\phi} & C_{26}^{\phi} & C_{66}^{\phi} \end{bmatrix} = det\begin{bmatrix} 45264 & 34064 & 33908 \\ 34064 & 45264 & 33908 \\ 33908 & 33908 & 36617 \end{bmatrix} = 6.7783 \cdot 10^{12}\ (MPa)^3$$

Elastic properties of lamina rotated by angle $\varphi = 45°$:

$$E_x = \frac{\Delta C}{C_{22}^{\varphi} \cdot C_{66}^{\varphi} - \left(C_{26}^{\varphi}\right)^2} = \frac{6.7783 \cdot 10^{12}}{45264 \cdot 36617 - (33908)^2} = 13552\ MPa$$

$$E_y = \frac{\Delta C}{C_{11}^{\varphi} \cdot C_{66}^{\varphi} - \left(C_{16}^{\varphi}\right)^2} = \frac{6.7783 \cdot 10^{12}}{45264 \cdot 36617 - (33908)^2} = 13352\ MPa$$

$$G_{xy} = \frac{\Delta C}{C_{11}^{\varphi} \cdot C_{22}^{\varphi} - \left(C_{12}^{\varphi}\right)^2} = \frac{6.7783 \cdot 10^{12}}{45264 \cdot 45264 - (34064)^2} = 7629\ MPa$$

$$\nu_{xy} = \frac{C_{12}^{\varphi} \cdot C_{66}^{\varphi} - C_{16}^{\varphi} \cdot C_{26}^{\varphi}}{C_{22}^{\varphi} \cdot C_{66}^{\varphi} - \left(C_{26}^{\varphi}\right)^2} = \frac{34064 \cdot 36617 - 33908 \cdot 33908}{45264 \cdot 36617 - (33908)^2} = 0.192$$

Similarly, the elastic characteristics of a lamina determines for other angels φ. The values of elastic and shear moduli depending on the angel φ in the polar coordinate system are shown in Fig. 8, and Poisson ratio – in Fig. 10. Also this values shown in the

Fabric

Secondary Poisson ratio:

$$\nu_{21} = \nu_{12} \cdot \frac{E_2}{E_1} = 0.07 \cdot \frac{63000}{65000} = 0.068.$$

Coefficients of stiffness matrix of lamina:

$$C_{11}^{0} = \frac{E_1}{1 - \nu_{12} \cdot \nu_{21}} = \frac{65000}{1 - 0.070 \cdot 0.068} = 65310\ MPa$$

$$C_{12}^{0} = \frac{E_1 \cdot \nu_{21}}{1 - \nu_{12} \cdot \nu_{21}} = \frac{E_2 \cdot \nu_{12}}{1 - \nu_{12} \cdot \nu_{21}} = 4431\ MPa$$

$$C_{22}^{0} = \frac{E_2}{1 - \nu_{12} \cdot \nu_{21}} = 63301\ MPa$$

$$C_{66}^{0} = G_{12} = 6500\ MPa$$

Independent coefficients:

$$V_1 = \frac{3 \cdot C_{11}^{0} + 2 \cdot C_{12}^{0} + 3 \cdot C_{22}^{0} + 4 \cdot C_{66}^{0}}{8}$$
$$= \frac{3 \cdot 65310 + 2 \cdot 4431 + 3 \cdot 63301 + 4 \cdot 6500}{8} = 52587\ MPa$$

$$V_2 = \frac{C_{11}^0 - C_{22}^0}{2} = \frac{65310 - 63301}{2} = 1005\ MPa$$

$$V_3 = \frac{C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 - 4 \cdot C_{66}^0}{8} = \frac{65310 - 2 \cdot 4431 + 63301 - 4 \cdot 6500}{8}$$
$$= 11719\ MPa$$

$$V_4 = \frac{C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 + 4 \cdot C_{66}^0}{8} = \frac{65310 - 2 \cdot 4431 + 63301 + 4 \cdot 6500}{8}$$
$$= 18219\ MPa$$

Coefficients of stiffness matrix of lamina rotated on angle $\varphi = 45°$:

$$C_{11}^{45} = V_1 + V_2 \cdot cos2\varphi + V_3 \cdot cos4\varphi$$
$$= 52587 + 1005 \cdot \cos(90°) + 11719 \cdot cos(180°) = 40868\ MPa$$

$$C_{12}^{\varphi} = V_1 - 2 \cdot V_4 - V_3 \cdot cos4\varphi = 52587 - 2 \cdot 18219 - 11719 \cdot cos(180°)$$
$$= 27868\ MPa$$

$$C_{16}^{\varphi} = 0.5 \cdot V_2 \cdot sin2\varphi + V_3 \cdot sin4\varphi = 0.5 \cdot 1005 \cdot sin(90°) + 11719 \cdot sin(180°)$$
$$= 502\ MPa$$

$$C_{22}^{\varphi} = V_1 - V_2 \cdot cos2\varphi + V_3 \cdot cos4\varphi$$
$$= 52587 - 1005 \cdot \cos(90°) + 11719 \cdot cos(180°) = 40868\ MPa$$

$$C_{26}^{\varphi} = 0.5 \cdot V_2 \cdot sin2\varphi - V_3 \cdot sin4\varphi = 0.5 \cdot 1005 \cdot sin(90°) - 11719 \cdot sin(180°)$$
$$= 502\ MPa$$

$$C_{66}^{\varphi} = V_4 - V_3 \cdot cos4\varphi = 18219 - 11719 \cdot cos(180°) = 29937\ MPa$$

Determinant of stiffness matrix:

$$\Delta C = det\begin{bmatrix} C_{11}^{\phi} & C_{12}^{\phi} & C_{16}^{\phi} \\ C_{12}^{\phi} & C_{22}^{\phi} & C_{26}^{\phi} \\ C_{16}^{\phi} & C_{26}^{\phi} & C_{66}^{\phi} \end{bmatrix} = det\begin{bmatrix} 40868 & 27868 & 502 \\ 27868 & 40868 & 502 \\ 502 & 502 & 29937 \end{bmatrix} = 2.67445 \cdot 10^{13}\ (MPa)^3$$

Elastic properties of lamina rotated by angle $\varphi = 45°$:

$$E_x = \frac{\Delta C}{C_{22}^{\varphi} \cdot C_{66}^{\varphi} - \left(C_{26}^{\varphi}\right)^2} = \frac{2.67445 \cdot 10^{13}}{40868 \cdot 29937 - (502)^2} = 21864\ MPa$$

$$E_y = \frac{\Delta C}{C_{11}^{\varphi} \cdot C_{66}^{\varphi} - \left(C_{16}^{\varphi}\right)^2} = \frac{2.67445 \cdot 10^{13}}{40868 \cdot 29937 - (502)^2} = 21864\ MPa$$

$$G_{xy} = \frac{\Delta C}{C_{11}^{\varphi} \cdot C_{22}^{\varphi} - \left(C_{12}^{\varphi}\right)^2} = \frac{2.67445 \cdot 10^{13}}{40868 \cdot 40868 - (27868)^2} = 29930\ MPa$$

$$\nu_{xy} = \frac{C_{12}^{\varphi} \cdot C_{66}^{\varphi} - C_{16}^{\varphi} \cdot C_{26}^{\varphi}}{C_{22}^{\varphi} \cdot C_{66}^{\varphi} - \left(C_{26}^{\varphi}\right)^2} = \frac{27868 \cdot 29937 - 502 \cdot 502}{40868 \cdot 29937 - (502)^2} = 0.682$$

Similarly, the elastic characteristics of a lamina determines for other angels φ. The values of elastic and shear moduli depending on the angel φ in the polar coordinate system are shown in Fig. 9, and Poisson ratio – in Fig. 11. Also this values shown in the Table 4.

Table 5 – Elastic properties of tape for different angels $\varphi°$

φ	E_x, MPa	E_y, Mpa	G_{xy}, Mpa	ν_{xy}
0	143000	8400	5600	0.360
15	57248	8733	5999	0.314
30	22773	10006	6995	0.260
45	13352	13352	7629	0.192
60	10006	22773	6995	0.114
75	8733	57248	5999	0.048
90	8400	143000	5600	0.021

Table 6 - Elastic properties of tape for different angels $\varphi°$

φ	E_x, MPa	E_y, MPa	G_{xy}, MPa	ν_{xy}
0	65000	63000	6500	0.070
15	43583	42795	8082	0.375
30	26255	26088	15743	0.621
45	21864	21864	29930	0.682
60	26088	26255	15743	0.617
75	42795	43583	8082	0.368
90	63000	65000	6500	0.068

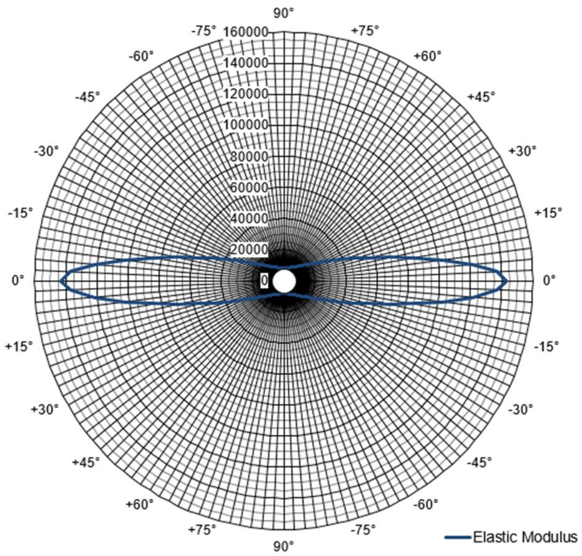


Fig. 13 Elastic modulus E of tape, depending on $\varphi°$ in polar coordinate system

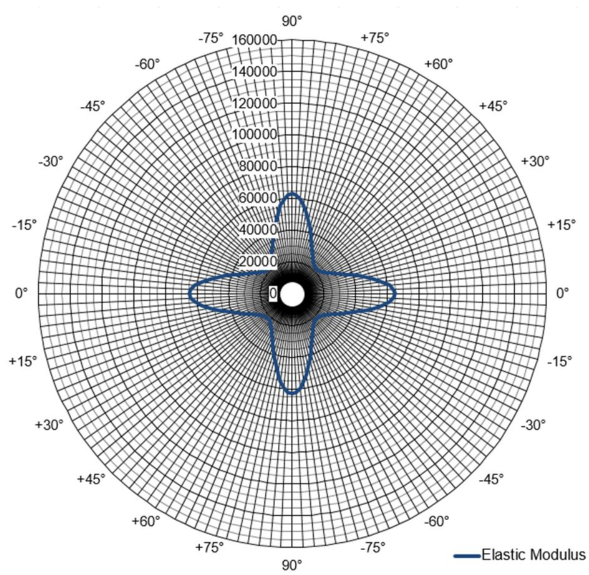


Fig. 14 Elastic modulus E of fabric, depending on $\varphi°$ in polar coordinate system

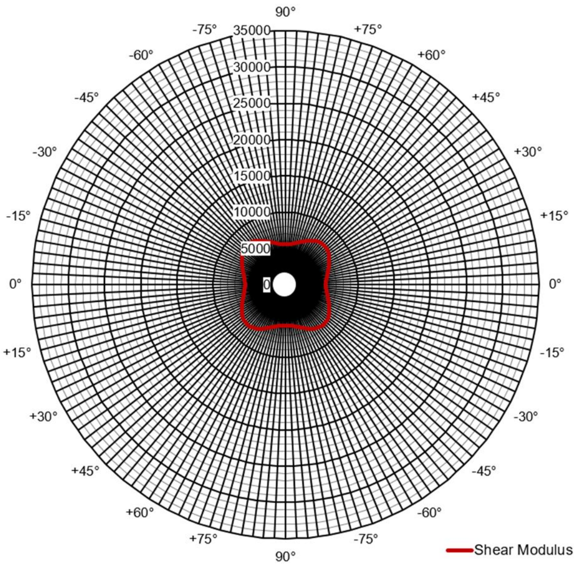


Fig. 15 Shear modulus G of tape, depending on φ° in polar coordinate system

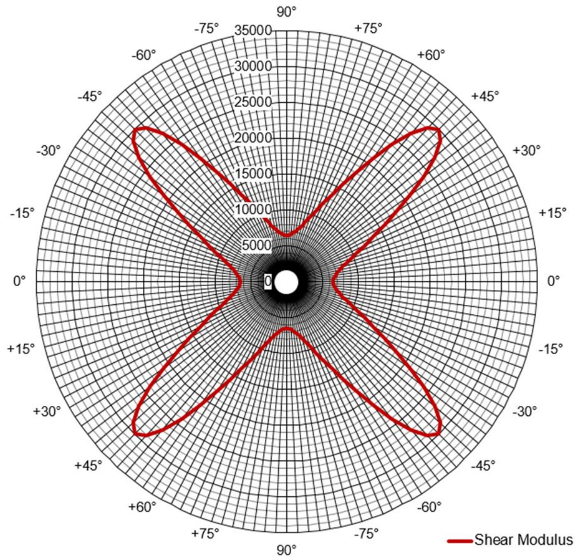


Fig. 16 Shear modulus G of fabric, depending on φ° in polar coordinate system

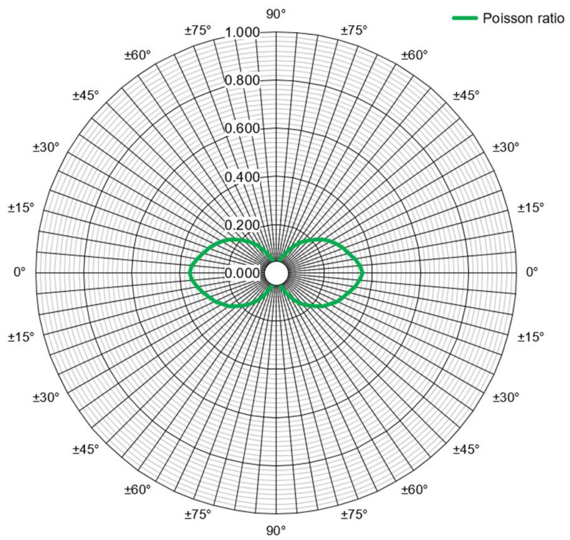


Fig. 17 Poisson ratio of tape, depending on φ° in polar coordinate system

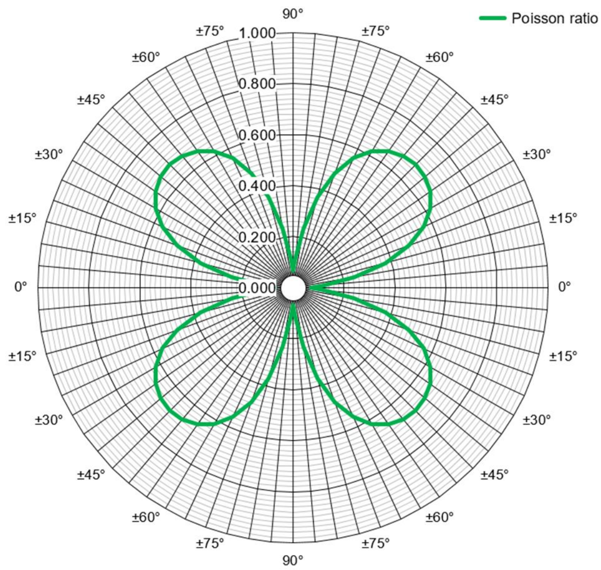


Fig. 18 Poisson ratio of fabric, depending on φ° in polar coordinate system

As can be seen tape has high elastic properties in one direction, thus, tape has higher capability to adjust properties of laminate for different directions.

2.2 ELASTIC PROPERTIES OF LAMINATE

For in-plane stress condition the stress-strain relationship for orthotropic laminate has same view like for orthotropic lamina:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2.37}$$

where C_{ij} components of stiffness matrix [6]:

$$C_{11} = \sum_{i=1}^{2n} C_{11}^{i} \cdot \frac{\delta_m}{s} \tag{2.38}$$

$$C_{12} = \sum_{i=1}^{2n} C_{12}^{i} \cdot \frac{\delta_m}{s} \tag{2.39}$$

$$C_{22} = \sum_{i=1}^{2n} C_{22}^{i} \cdot \frac{\delta_m}{s} \tag{2.40}$$

$$C_{66} = \sum_{i=1}^{2n} C_{66}^{i} \cdot \frac{\delta_m}{s} \tag{2.41}$$

δ_m – thickness of lamina;

s – thickness of laminate;

n – number of laminas relatively to mid plane.

Obviously, the alternation order of laminas in the laminate doesn't matter for determining elastic properties of laminate.

Elastic properties of orthotropic laminate determined by the following equations [6]:

$$E_x = C_{11} - \frac{C_{12}^2}{C_{22}} \tag{2.42}$$

$$E_x = C_{22} - \frac{C_{12}^2}{C_{11}} \tag{2.43}$$

$$G_{xy} = C_{66} \tag{2.44}$$

$$\nu_{xy} = \frac{C_{12}}{C_{22}} \tag{2.45}$$

ν_{xy} – determines by Maxwell ratio:

$$\nu_{xy} \cdot E_y = \nu_{yx} \cdot E_x \tag{2.46}$$

Thus, behavior of laminate in for in-plane stress condition describes same as for lamina by four independent elastic constants.

Vector of deformations can be written by following form [2]:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_x^{\mathrm{o}} \\ \epsilon_y^{\mathrm{o}} \\ \gamma_{xy}^{\mathrm{o}} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.47}$$

where ϵ_x^o, ϵ_y^o, γ_{xy}^o are the strains in the middle plane, and κ_x^0, κ_y^0, κ_{xy}^0 are the curvatures of the reference plane of the laminate.

The in-plane forces and moments acting om a small element are [2]:

$$N_x = \int_{-h_b}^{h_t} \sigma_x dz \tag{2.48}$$

$$N_y = \int_{-h_b}^{h_t} \sigma_y dz \tag{2.49}$$

$$N_{xy} = \int_{-h_b}^{h_t} \tau_{xy} dz \tag{2.50}$$

$$M_x = \int_{-h_b}^{h_t} z \cdot \sigma_x dz \tag{2.51}$$

$$M_y = \int_{-h_b}^{h_t} z \cdot \sigma_y dz \tag{2.52}$$

$$M_{xy} = \int_{-h_b}^{h_t} z \cdot \tau_{xy} dz \tag{2.53}$$

After integration of Eqs. (2.48-2.53), the expressions for the in-plane forces and moments become [2]:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^{\circ} \\ \epsilon_y^{\circ} \\ \gamma_{xy}^{\circ} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.54}$$

In the Eq. (2.54) the stiffness matrices [*A*], [*B*] and [*D*] can be highlighted [2], where:

$$A_{ij} = \sum_{k=1}^{2n} (C_{ij}^{\varphi})_k \cdot (z_k - z_{k-1}) \tag{2.55}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{2n} (C_{ij}^{\varphi})_k \cdot (z_k^2 - z_{k-1}^2) \tag{2.56}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{2n} (C_{ij}^{\varphi})_k \cdot (z_k^3 - z_{k-1}^3) \tag{2.57}$$

Where 2n is the total number of plies (or ply groups) in the laminate; z_k, z_{k-1} are the distance from the middle plane to the two surfaces of the k-th play: and $(C_{ij}^{\varphi})_k$ are the element of the stiffness matrix of the k-ht ply.

2.3 THE SIGNIFICANCE OF THE [*A*], [*B*] AND [*D*] STIFFNESS MATRICES

The [*A*], [*B*] and [*D*] matrices represent the stiffnesses of a laminate and describe the response of the laminate to in-plane forces and moments.

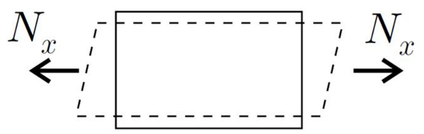
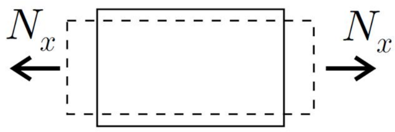
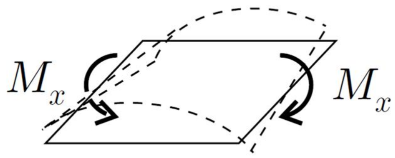
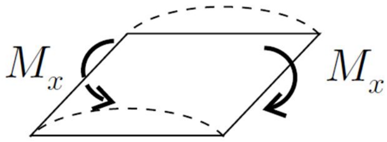
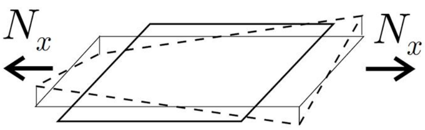
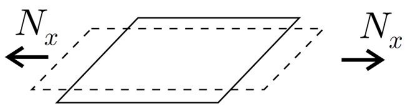
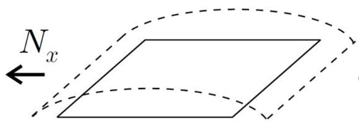
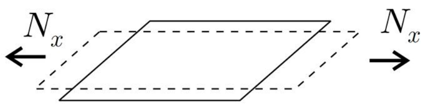
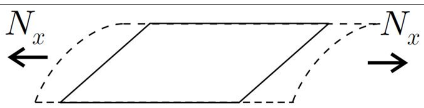
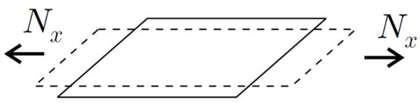
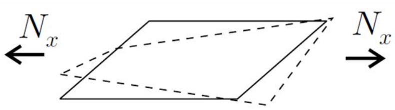
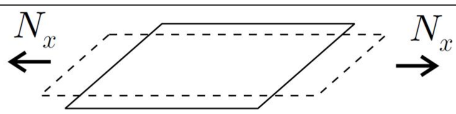
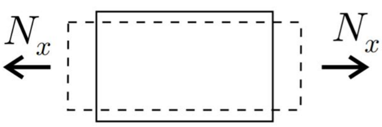
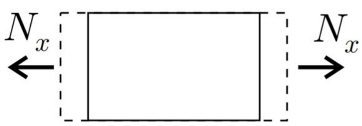
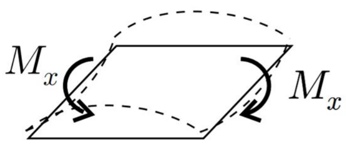
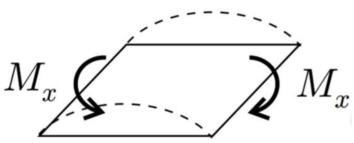
A_{ij} are the in-plane stiffnesses that relate the in-plane forces N_x, N_y, N_{xy} to the in-plane deformations ϵ_x^0, ϵ_y^0, γ_{xy}^0.

D_{ij} are the bending stiffnesses that relate the moments M_x, M_y, M_{xy} to the curvatures κ_x^0, κ_y^0, κ_{xy}^0.

B_{ij} are the in-plane-out-of-plane coupling stiffnesses that relate the in-plane forces N_x, N_y, N_{xy} to the curvatures κ_x^0, κ_y^0, κ_{xy}^0 and the moments M_x, M_y, M_{xy} to the in-plane deformations ϵ_x^0, ϵ_y^0, γ_{xy}^0.

The Table 4 shows relationships between elements of stiffness matrices and deform shapes for different loads.

Table 7 – Illustration of the coupling terms for composite materials

Coupling	No Coupling	Element
Extension – shear 		A_{16}
Bending – twist 		D_{16}
Extension – twist 		B_{16}
In-plane-out-of-plane 		B_{11}
		B_{12}
		B_{66}
Extension-extension 		A_{12}
Bending-bending 		D_{12}

Symmetrical laminate. In a symmetrical laminate the ply located at a position +z is identical to the ply at −z. Correspondingly, the stiffness matrix [C] of the ply at +z is identical to the stiffness matrix of the ply at −z.

Balanced laminate. In a balanced laminate, for every unidirectional ply in the + θ direction (measured counterclockwise from the x coordinate) there is an identical ply in the -θ.

Orthotropic laminate. In orthotropic laminates we are interested in two mutually perpendicular directions, called orthotropy directions, in the plane of the laminate. Normal forces and bending moments applied in these directions do not cause shear or twist of the laminate. Hence, there are no extension–shear, bending–twist, and extension–twist couplings.

Table 8 – The [A], [B], [D] matrices for laminates

[A]	[B]	[D]
Symmetrical $\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}$
Balanced $\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$	$\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}$
Orthotropic $\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$	$\begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}$
Isotropic $\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & \frac{A_{11}-A_{12}}{2} \end{bmatrix}$	$\begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{11} & 0 \\ 0 & 0 & \frac{B_{11}-B_{12}}{2} \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{11} & 0 \\ 0 & 0 & \frac{D_{11}-D_{12}}{2} \end{bmatrix}$

3 CHOOSE OF THE PLIES SEQUENCE

The choose of plies sequence depends from loaded condition of designed structure. The laminate must be stiffed in the load path direction. For example, for basically compression loaded structure the longitudinal plies must be as far apart from the middle plane as possible and for shear loaded structure plies with ±45º orientation must be as far apart from the middle plane as possible. On figure below, effect of stacking is shown.

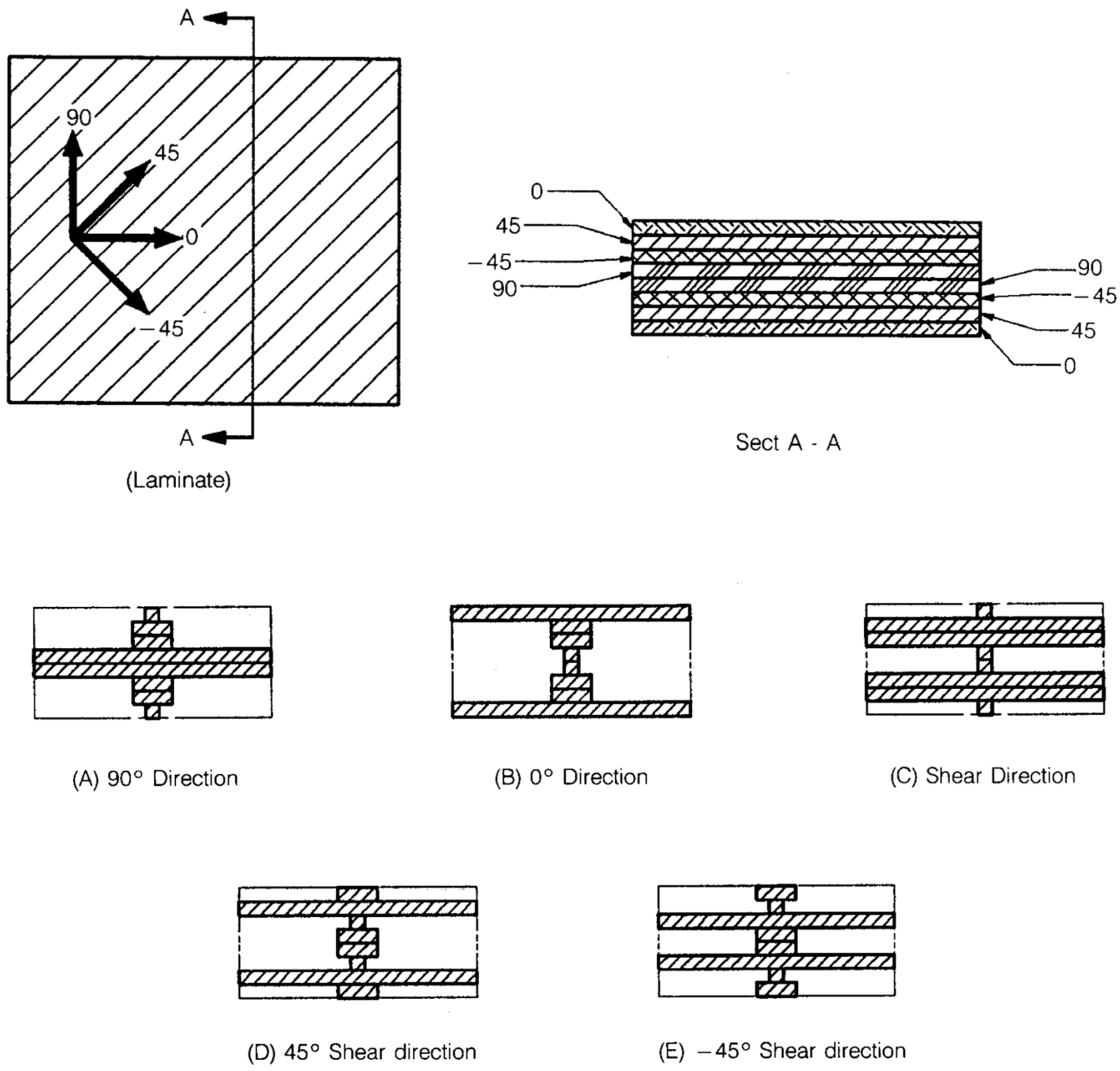


Fig. 19 Effect of Stacking Sequence [1]

The panels of wing basically loaded by compression or tension, so laminate must to have longitudinal plies near to top and bottom of panel. Also since wing panels loaded by axial loads for panel material will be use symmetrical laminate to prevent all out of plane effects.

Laminate can be manufactured by many methods. Two main methods are lay-up and winding and plies can be separated by symmetry plane on two parts and space between this two plates filled by core to increase moment of inertia of panel.

For lay-up, ESDU standard library of sequences of plies can be used. ESDU document provides many different sequences to get different types of laminates (symmetrical, balanced or orthotropic). For analysis, sequences with maximum amount of ±45° plies will be used. On the fig. 9 shown few symmetric sequences from ESDU.

S 30	+*--**+-+*	*+-+**--*+	20	12	8	5040	2960	0.600	0.630
S 31	+*-*--++-+	+-++--*-*+	20	16	4	5248	2752	0.800	0.656
S 32	+*--*+-**+	+**-+*--*+	20	12	8	5328	2672	0.600	0.666
S 33	+-*-*+*+-*	*-+*+*-*-+	20	12	8	5616	2384	0.600	0.702
S 34	+-**-+-+-+	+-+-+-**-+	20	16	4	5632	2368	0.800	0.704
S 35	+-*-+**-*+	+*-**+-*-+	20	12	8	5808	2192	0.600	0.726
S 36	+-*-*++--+	+--++*-*-+	20	16	4	5920	2080	0.800	0.740
S 37	+-*-+*-+**	**+-*+-*-+	20	12	8	6096	1904	0.600	0.762
S 38	+*---+++*-	-*+++---*+	20	16	4	6208	1792	0.800	0.776
S 39	+--+***-+*	*+-***+--+	20	12	8	6480	1520	0.600	0.810
S 40	+--*+*+-+-	-+-+*+*--+	20	16	4	6496	1504	0.800	0.812
S 41	+-*+---*++	++*---+*-+	20	16	4	6496	1504	0.800	0.812
S 42	+-*-+-+*+-	-+*+-+-*-+	20	16	4	6496	1504	0.800	0.812
S 43	+--*++-***	***-++*--+	20	12	8	6768	1232	0.600	0.846
S 44	+--+**-++-	-++-**+--+	20	16	4	6784	1216	0.800	0.848
S 45	+--+*-++-*	*-++-*+--+	20	16	4	7264	736	0.800	0.908
S 46	+--+-+*+*-	-*+*+-+--+	20	16	4	7648	352	0.800	0.956
S 47	+-+---++**	**++---+-+	20	16	4	7936	64	0.800	0.992
S 48	+--+-++-**	**-++-+--+	20	16	4	7936	64	0.800	0.992

Fig. 20 Standard symmetric sequences [4]

where

+ - layer is aligned at + angle φ to x-axis;

- - layer is aligned at - angle φ to x-axis;

* - layer is aligned with either x- or y-axis.

For analysis, the following sequences of plies were selected: S31, S34, S36, S38, S40, S41, S42, S44, S45, S46, S47 and S48.

The winding laminates consist from pairs of layers with opposite angels. For analysis, twenty ply symmetric laminates will be used with following rules of plies orientation: ±30°, ±45° and ±60°.

For honeycomb panel, the sequence S60 will be used.

```
S 61   +**---*+*+ * +*+*---**+     21     12     9    5292    3969    0.571    0.571
```

Fig. 21 Sequence number S60 [4]

4 BUCKLING OF LAMINATES

Buckling occurs in the structures or in the separate elements of structure, which loaded by compression or shear loads or under interaction of compression and shear. Capability of resistance buckling defines by bending or torsional stiffness of loaded element and by elastic properties of material.

In this work rectangular simply supported composite plates will be analyze and will be determined the most optimal variant of laminate.

4.1 BUCKLING UNDER UNIDIRECTIONAL LOAD

Critical load for simply supported and uniaxial loaded plate defines by following equation [2]:

$$N_{x.cr} = \frac{\pi^2}{L_y^2} \cdot \left(D_{11} \frac{L_y^2}{l_x^2} + D_{22} \frac{l_x^2}{L_y^2} + 2(D_{12} + 2D_{66})\right) \tag{4.1}$$

where

L_y – width of plate;

l_x – length of half buckling wave (Fig. 17).

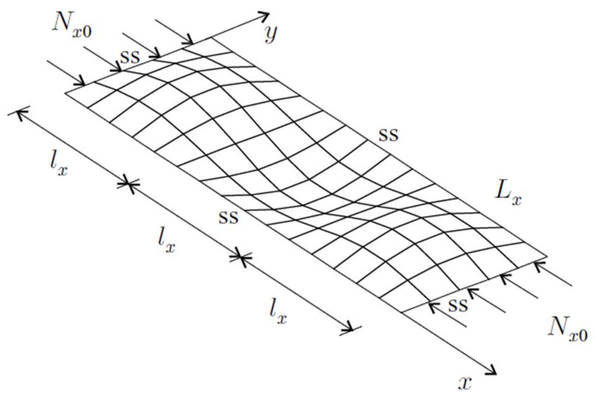


Fig. 22 Buckled shape of a uniaxial loaded rectangular plate with simply supported edges [2]

Obviously, that the minimum critical corresponds to one half buckling wave. Thus l_x is equal to length of plate L_x which is taken equal width $l_x = L_y$ and Equ. 4.1 takes following form:

$$N_{x.cr} = \frac{\pi^2}{L_y^2} \cdot (D_{11} + D_{22} + 2(D_{12} + 2D_{66})) \tag{4.2}$$

4.2 BUCKLING UNDER SHEAR LOAD

Buckling in the shear structure occurs due third principal stress, which directed at an angle of 45° to direction of shear stress.

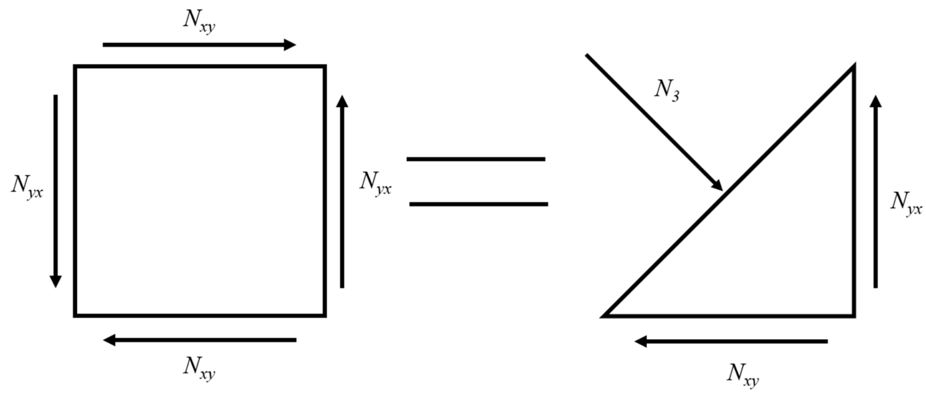


Fig. 23 Pure shear

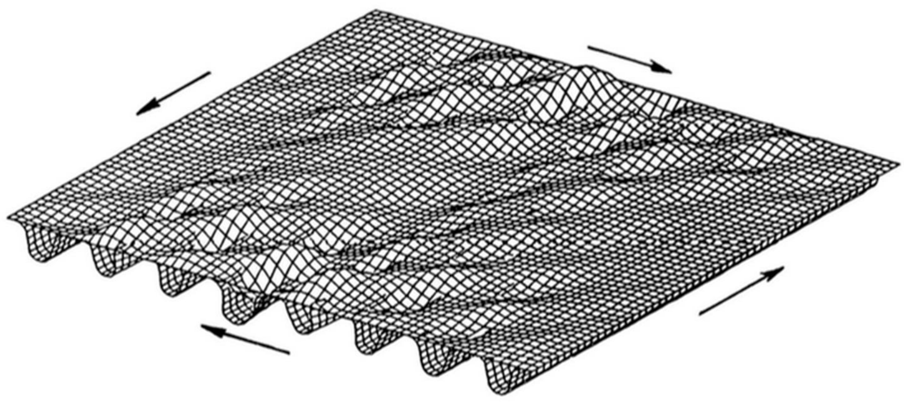


Fig. 24 Buckled form of shear loaded panel with stiffeners

The critical shear load for simply supported rectangular plate determines by following equation [2]:

$$N_{xy.cr} = \begin{cases} \dfrac{4}{L_y^2}\sqrt[4]{D_{11}D_{22}^3}\,(8.125 + 5.045K) & 0 \le K \le 1 \\ \dfrac{4}{L_y^2}\sqrt{D_{22}(D_{12} + 2D_{66})}\,(11.71 + \dfrac{1.46}{K^2}) & 1 \le K \le \infty \end{cases} \tag{4.3}$$

$$K = \frac{2D_{66} + D_{12}}{\sqrt{D_{11}D_{22}}} \tag{4.4}$$

where

K – stiffness parameter;

The shear buckling capability of laminate can be improved by increasing number of ±45° plies and by moving them as far away from the mid-surface as possible.

5. DETERMINATION OF STIFFNESS PROPERTIES OF LAMINAT

All values on next pages were calculated by equations provides in the sections 2 and 4. Detail analysis for lay-up laminate with plies sequence S31 shown in the appendix A. Analysis for other laminates were done in a similar manner. Since there are big set of data for all analyzed panels, this values was not shown in this dissertation.

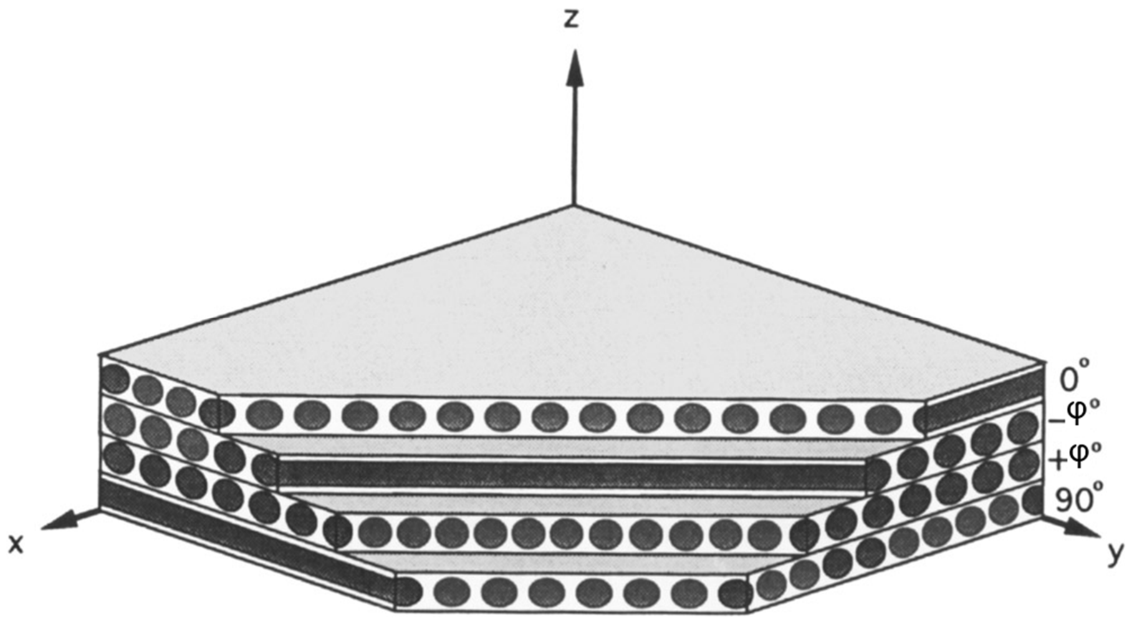


Fig. 25 Example of ply sequence

All considered panels except sandwich panels have 20 plies with thickness of 0.125 mm.

5.1 LAY-UP LAMINATES

Table 9 – Stiffness properties and critical loads for laminates from tape plies with * plies oriented along laminate.

* - 0°	D_{11}	D_{12}	D_{22}	D_{66}	$N_{x.cr}$	K	$N_{xy.cr}$
S31	103206	30461	42454	33785	337.27	1.48	319.35
S34	97029	32400	44754	35724	344.92	1.58	335.36
S36	92397	33854	46479	37178	350.66	1.65	347.37
S38	87764	35308	48204	38632	356.40	1.73	359.40
S40	83131	36761	49929	40085	362.14	1.81	371.45
S41	83131	36761	49929	40085	362.14	1.81	371.45
S42	83131	36761	49929	40085	362.14	1.81	371.45
S44	78498	38215	51654	41539	367.88	1.90	383.50
S45	70777	40639	54529	43963	377.45	2.07	403.60
S46	64600	42577	56829	45901	385.10	2.22	419.70
S47	59967	44031	58554	47355	390.84	2.34	431.78
S48	59967	44031	58554	47355	390.84	2.34	431.78

Table 10 – Stiffness properties and critical loads for laminates from fabrics plies with * plies oriented along laminate.

* - 0°	D_{11}	D_{12}	D_{22}	D_{66}	$N_{x.cr}$	K	$N_{xy.cr}$
S31	64162	25789	63262	28483	289.11	1.30	363.95
S34	62634	27254	61860	29948	294.90	1.40	365.79
S36	61488	28352	60808	31046	299.23	1.48	367.16
S38	60343	29451	59757	32145	303.57	1.56	368.51
S40	59197	30550	58705	33243	307.91	1.65	369.79
S41	59197	30550	58705	33243	307.91	1.65	369.79
S42	59197	30550	58705	33243	307.91	1.65	369.79
S44	58051	31648	57654	34342	312.24	1.73	371.01
S45	56142	33479	55901	36173	319.47	1.89	372.85
S46	54614	34944	54499	37638	325.26	2.02	374.12
S47	53468	36043	53448	38737	329.59	2.12	374.93
S48	53468	36043	53448	38737	329.59	2.12	374.93

Table 11 - Stiffness properties and critical loads for laminates from tape plies with * plies oriented across laminate.

* - 90°	$D_{11}, N \cdot mm$	$D_{12}, N \cdot mm$	$D_{22}, N \cdot mm$	$D_{66}, N \cdot mm$	$N_{x.cr}, \frac{N}{mm}$	K	$N_{xy.cr} \frac{N}{mm}$
S31	42454	30461	103206	33785	337.27	1.48	497.93
S34	44754	32400	97029	35724	344.92	1.58	493.79
S36	46479	33854	92397	37178	350.66	1.65	489.77
S38	48204	35308	87764	38632	356.40	1.73	484.95
S40	49929	36761	83131	40085	362.14	1.81	479.29
S41	49929	36761	83131	40085	362.14	1.81	479.29
S42	49929	36761	83131	40085	362.14	1.81	479.29
S44	51654	38215	78498	41539	367.88	1.90	472.76
S45	54529	40639	70777	43963	377.45	2.07	459.82
S46	56829	42577	64600	45901	385.10	2.22	447.48
S47	58554	44031	59967	47355	390.84	2.34	436.96
S48	58554	44031	59967	47355	390.84	2.34	436.96

Table 12 - Stiffness properties and critical loads for laminates from fabric plies with * plies oriented across laminate.

* - 90°	$D_{11}, N \cdot mm$	$D_{12}, N \cdot mm$	$D_{22}, N \cdot mm$	$D_{66}, N \cdot mm$	$N_{x.cr}, \frac{N}{mm}$	K	$N_{xy.cr} \frac{N}{mm}$
S31	63262	25789	64162	28483	289.11	1.30	366.53
S34	61860	27254	62634	29948	294.90	1.40	368.07
S36	60808	28352	61488	31046	299.23	1.48	369.21
S38	59757	29451	60343	32145	303.57	1.56	370.31
S40	58705	30550	59197	33243	307.91	1.65	371.34
S41	58705	30550	59197	33243	307.91	1.65	371.34
S42	58705	30550	59197	33243	307.91	1.65	371.34
S44	57654	31648	58051	34342	312.24	1.73	372.29
S45	55901	33479	56142	36173	319.47	1.89	373.65
S46	54499	34944	54614	37638	325.26	2.02	374.51
S47	53448	36043	53468	38737	329.59	2.12	375.01
S48	53448	36043	53468	38737	329.59	2.12	375.01

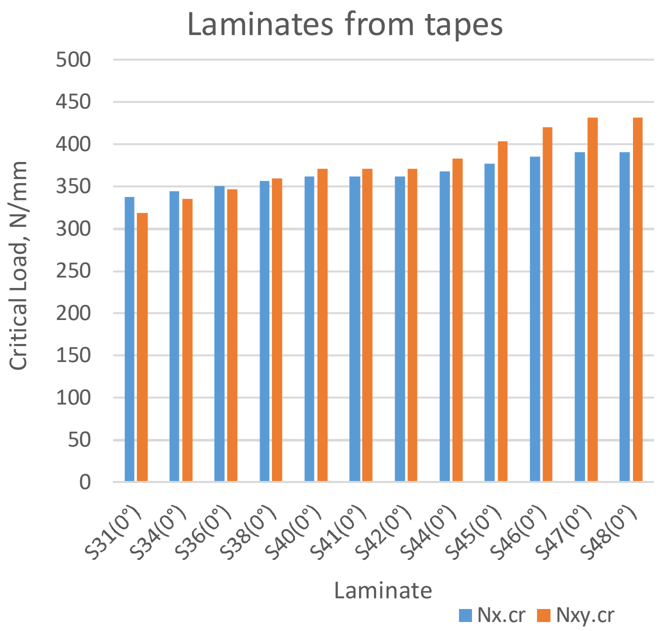


Fig. 26 Critical loads for laminates from tapes with * plies along laminate

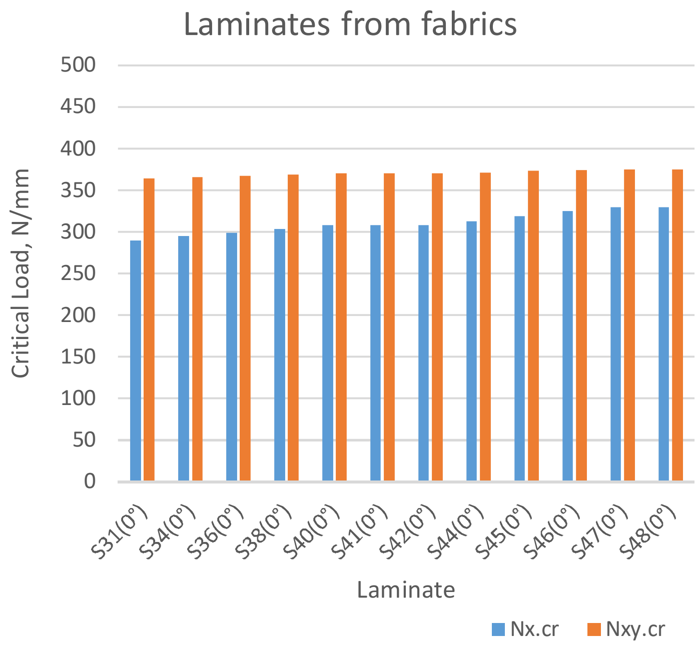


Fig. 27 Critical loads for laminates from fabrics with * plies along laminate

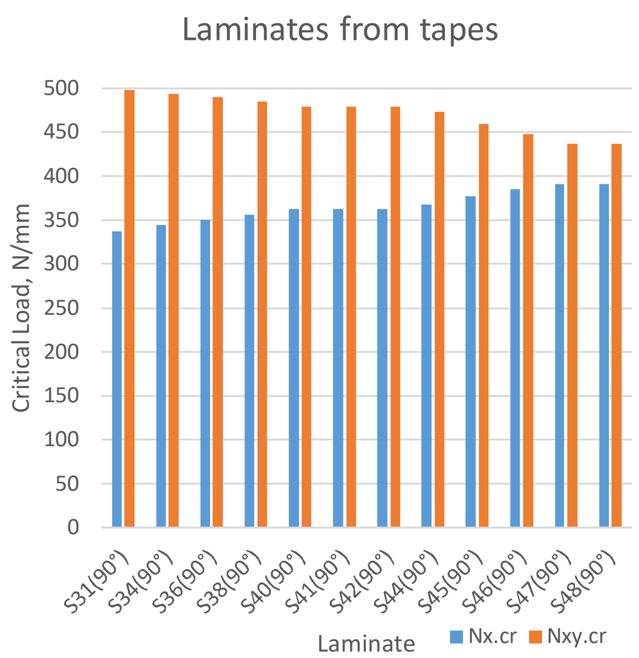


Fig. 28 Critical loads for laminates from tapes with * plies across laminate

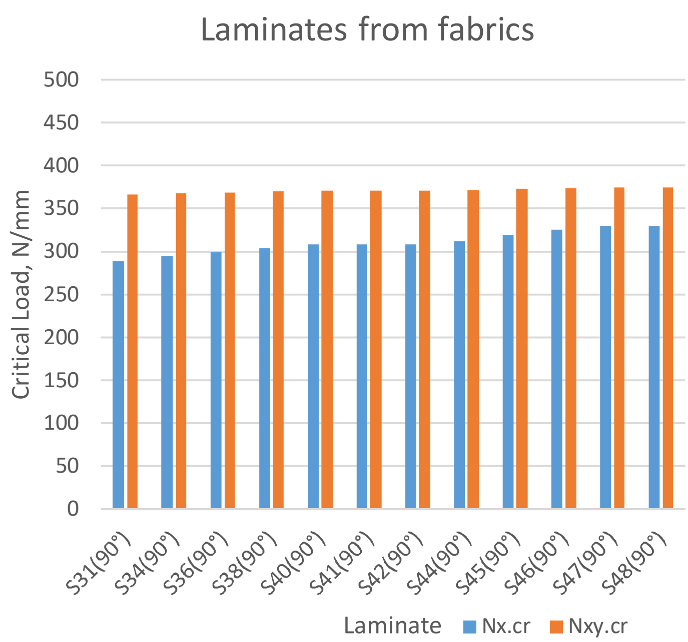


Fig. 29 Critical loads for laminates from fabrics with * plies across laminate

5.2 WINDING LAMINATES

Table 13 – Stiffness properties and critical of winding laminates

* ±φ°	$D_{11}, N \cdot mm$	$D_{12}, N \cdot mm$	$D_{22}, N \cdot mm$	$D_{66}, N \cdot mm$	$N_{x.cr}, \frac{N}{mm}$	K	$N_{xy.cr} \frac{N}{mm}$
±30°	113185	34258	24883	37581	352.26	2.06	251.58
±45°	58937	44354	58937	47678	392.11	2.37	434.47
±60°	24883	34258	113185	37581	342.15	2.06	536.55

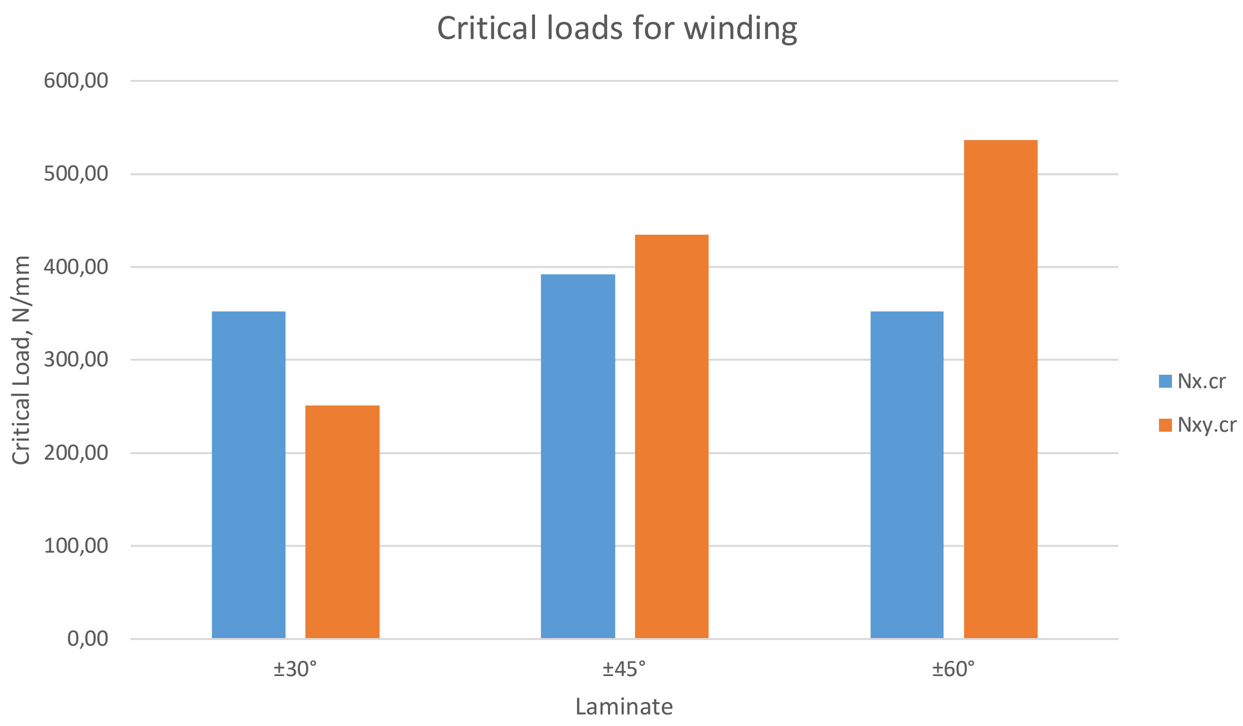


Fig. 30 Critical loads for winding laminates

5.3 SANDWICH PANELS

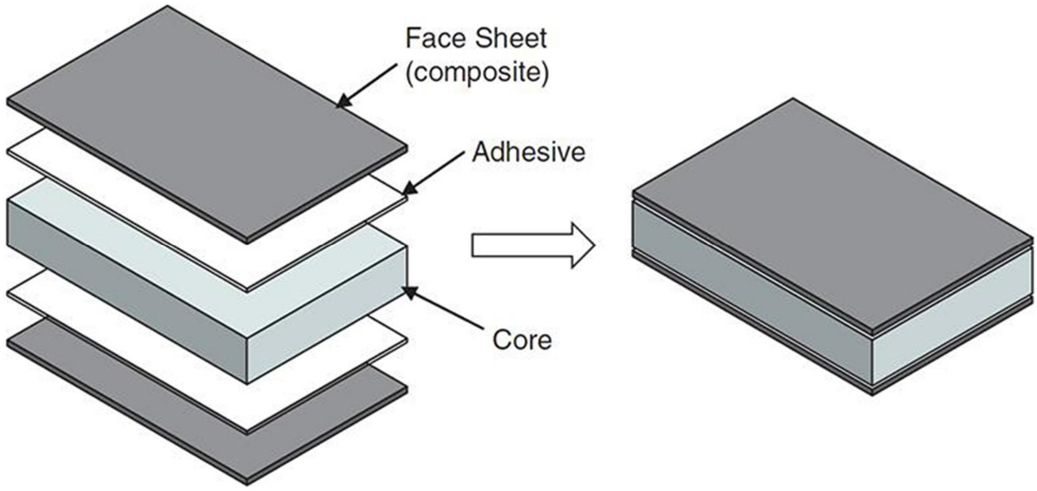


Fig. 31 Layers of sandwich panel

For face sheets were considered two options of sequence: with * plies directed by 0° and 90°. Face sheets of analyzed sandwich panel have following sequences of plies:

Table 14 – Plies sequence of face sheets of sandwich panel

Ply	Angle φ°
1	45
2	0/90
3	-45
4	0/90
5	0/90
6	-45
7	-45
8	0/90
9	45
10	45

Opposite face sheet has symmetrical sequence of plies to sequence shown in the Table 14.

In analysis were considered sandwich panels with different thickness of core. The core with thickness of one ply of lamina is ply of resin with micro balls. This ply used to little bit increase moment of inertia of panel. Thicker cores can be produced from honeycomb structure or from foam.

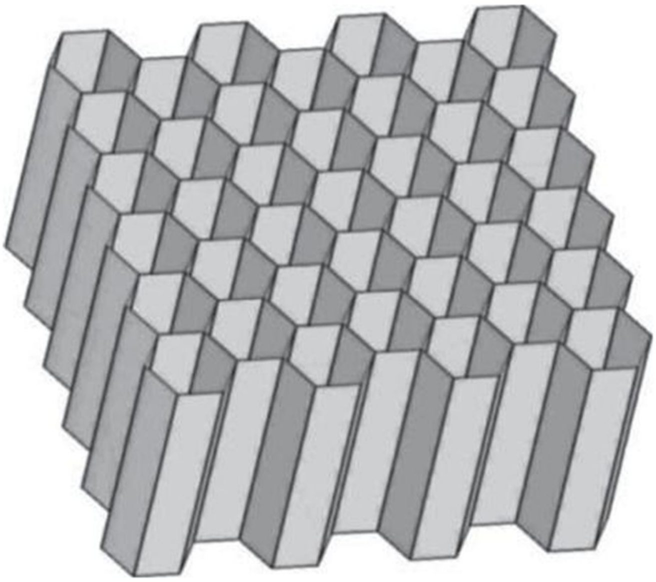


Fig. 32 Honeycomb core

Since core has significant lower stiffness for axial loads, assume that core has unit stiffness and just is increased moment of inertia of plies. It is conservative assumption.

Table 15 - Stiffness properties and critical loads for three plies panel which has composite face sheets from tapes with * plies oriented along laminate.

* - 0°	t, mm	$D_{11}, N \cdot mm$	$D_{12}, N \cdot mm$	$D_{22}, N \cdot mm$	$D_{66}, N \cdot mm$	$N_{x.cr}, \frac{N}{mm}$	K	$N_{xy.cr} \frac{N}{mm}$
S60	1 ply	135934	30097	43018	33945	370.01	1.28	327.18
	1	312122	70486	100549	79394	859.86	1.29	764.13
	10	6834880	1688914	2389029	1891758	19905.78	1.35	18087.61
	20	24152545	6058971	8559393	6780931	69260.09	1.36	56893.89

Table 16 - Stiffness properties and critical loads for three plies panel which has composite face sheets from fabrics with * plies oriented along laminate.

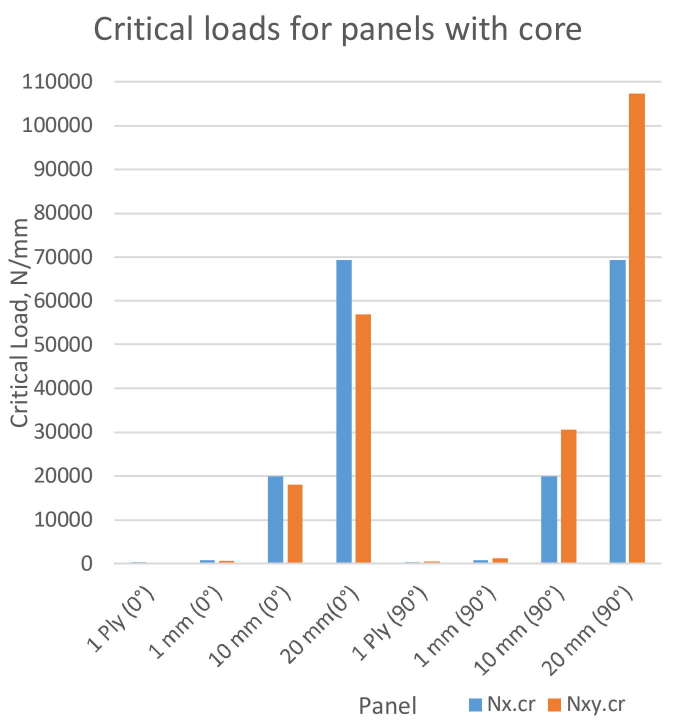
* - 0°	t, mm	$D_{11}, N \cdot mm$	$D_{12}, N \cdot mm$	$D_{22}, N \cdot mm$	$D_{66}, N \cdot mm$	$N_{x.cr}, \frac{N}{mm}$	K	$N_{xy.cr} \frac{N}{mm}$
S60	1 ply	78335	25950	76959	29068	319.25	1.08	416.85
	1	180740	60689	177605	67909	741.56	1.10	965.75
	10	4047301	1445253	3981431	1609667	17131.57	1.16	22049.54
	20	14358065	5179749	5764997	6780931	56855.12	2.06	50118.30

Table 17 - Stiffness properties and critical loads for three plies panel which has composite face sheets from tapes with * plies oriented across laminate.

* - 90°	t, mm	$D_{11}, N \cdot mm$	$D_{12}, N \cdot mm$	$D_{22}, N \cdot mm$	$D_{66}, N \cdot mm$	$N_{x.cr}, \frac{N}{mm}$	K	$N_{xy.cr} \frac{N}{mm}$
S60	1 ply	135934	30097	43018	33945	370.01	1.28	581.57
	1	312122	70486	100549	79394	859.86	1.29	1346.29
	10	6834880	1688914	2389029	1891758	19905.78	1.35	30593.98
	20	24152545	6058971	8559393	6780931	69260.09	1.36	107374.81

Table 18 - Stiffness properties and critical loads for three plies panel which has composite face sheets from fabrics with * plies oriented across laminate.

* - 90°	t, mm	$D_{11}, N \cdot mm$	$D_{12}, N \cdot mm$	$D_{22}, N \cdot mm$	$D_{66}, N \cdot mm$	$N_{x.cr}, \frac{N}{mm}$	K	$N_{xy.cr} \frac{N}{mm}$
S60	1 ply	76959	25950	76959	29068	317.89	1.09	416.15
	1	177605	60689	180740	67909	741.56	1.10	974.23
	10	3981431	1445253	4047301	1609667	17131.57	1.16	22231.19
	20	5764997	5179749	14358065	6780931	56855.12	2.06	79094.22



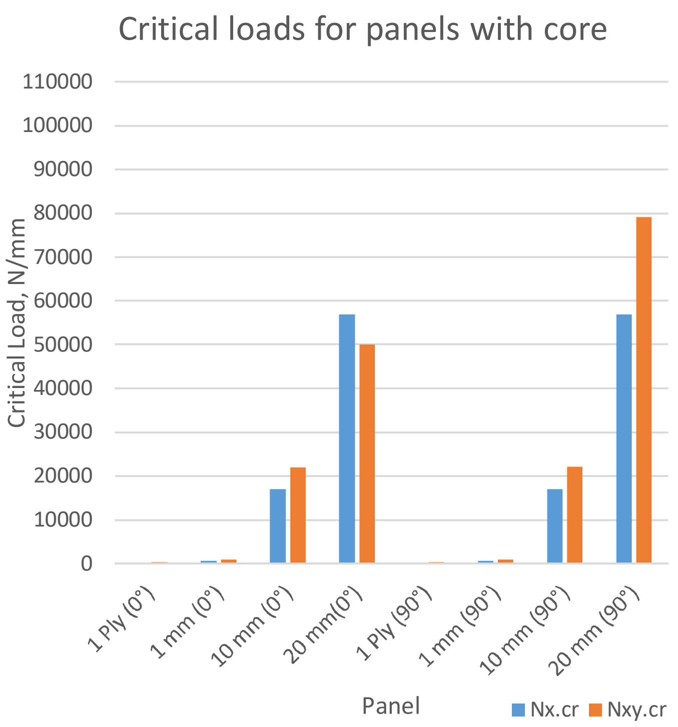


Fig. 33 Critical loads for panels with core and composite face sheets from tape plies

Fig. 34 Critical loads for panels with core and composite face sheets from fabric plies

As can be seen core can significant increase critical load of panel. Thus, sandwich panels can be used in the structure without additional reinforcement, but if it's required sandwich panels can be also reinforced by stiffeners to further increase the critical load.

5.4 REINFORCED LAY-UP PANELS

The reinforcing elements like stringers increase moment of inertia of panel and split the panel into separated elements, which have higher local buckling capability. The stringers should be matched so that local buckling critical load is equal to critical for entire panel. Let number of stringers of three. Thus, stringers split panel into four parts. Like panel was used lay-up laminate with plies sequence S31.

The critical load for entire panel determines by following equating:

$$N_{x.cr} = \frac{\pi^2}{L_y^2} \cdot \left(D'_{11} + D_{22} + 2(D_{12} + 2D_{66})\right) \tag{5.1}$$

where

$N_{x.cr}$- local buckling critical load;

$D_{11}{}'$- bending stiffness considering stringers, which determines by Equ. 5.2 [5].

$$D'_{11} = D_{11} + \frac{E'J}{d} \tag{5.2}$$

where

D_{11}- bending stiffness of panel;

E'- elastic modulus of stringer;

J- moment of inertia of stringer about the axis in middle plane of panel;

d- distance between stringers.

$$d = \frac{L_y}{n+1} = \frac{400}{3+1} = 100\ mm$$

where

n- number of stringers.

The local buckling critical load calculates by Equ. 4.1.

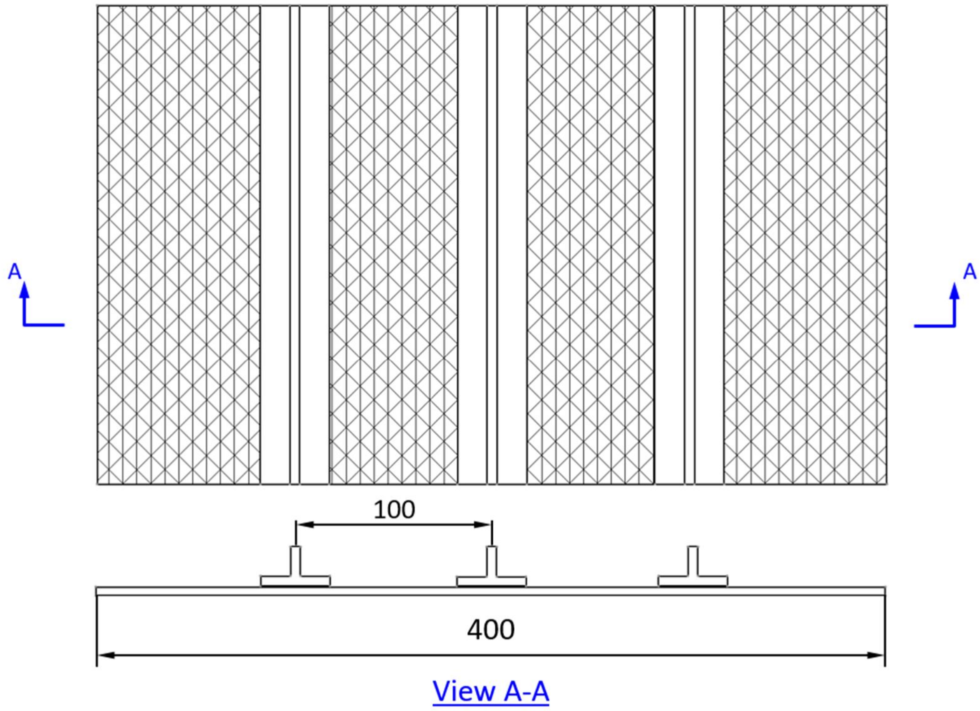


Fig. 35 Reinforced panel

Assume that stringer manufactured by pultrusion and elastic modulus of it is equal to elastic modulus of tape.

The most efficient reinforced panel has same critical load of local buckling between stringers and critical load of entire panel.

The local buckling critical loads are same for laminates with plies aligned by 0° and 90° (see Tables 9-12).

$N_{x.cr}^{local} = 337.27\frac{N}{mm}$ – critical load for laminate from tapes;

$N_{x.cr}^{local} = 289.11\frac{N}{mm}$ – critical load for laminate from fabrics.

Obtaining stiffness of panel with stringers and moment of inertia of stringers are obtained by transforming Equ. 5.1 and Equ. 5.2 respectively.

$$D'_{11} = \left(N_{x.cr}^{local}\frac{L_y^2}{\pi^2} - D_{22} - 2(D_{12} + 2D_{66})\right)$$

$$J = (D'_{11} - D_{11}) \cdot \frac{d}{E'}$$

Reinforced Panel from Tapes

$$D'_{11} = \left(N_{x.cr}^{local}\frac{L_y^2}{\pi^2} - D_{22} - 2(D_{12} + 2D_{66})\right)$$

$$= \left(337\frac{400^2}{3.14^2} - 42454 - 2(30461 + 2 \cdot 33785)\right) = 5229062\ N \cdot mm$$

$$J = (D'_{11} - D_{11}) \cdot \frac{d}{E'} = (5229062 - 103206) \cdot \frac{100}{143000} = 3585\ mm^4$$

Reinforced Panel from Fabrics Plies

$$D'_{11} = \left(N_{x.cr}^{local}\frac{L_y^2}{\pi^2} - D_{22} - 2(D_{12} + 2D_{66})\right)$$

$$= \left(289\frac{400^2}{3.14^2} - 63262 - 2(25789 + 2 \cdot 28483)\right) = 4457247\ N \cdot mm$$

$$J = (D'_{11} - D_{11}) \cdot \frac{d}{E'} = (4457247 - 64162) \cdot \frac{100}{143000} = 6760\ mm^4$$

For preliminary analysis, it is acceptable to use Equ. 5.2 for moment of inertia if stringer, but in further design analysis the moment of inertia should be adjusted by successive approximation method.

The moments of inertia for reinforced panel with * plies by 90° calculated by same way and values obtain same.

Table 19 - Stiffness properties and critical loads for reinforced panels from tape plies.

* - 0°	$D_{11}, N \cdot mm$	$D_{12}, N \cdot mm$	$D_{22}, N \cdot mm$	$D_{66}, N \cdot mm$	$N_{x.cr}, \frac{N}{mm}$
S31	103206	30461	42454	33785	337.27
S31+stringer	5229062	30461	42454	33785	337.27
* - 90°					
S31	42454	30461	103206	33785	337.27
S31+stringer	5168310	30461	103206	33785	337.27

Table 20 - Stiffness properties and critical loads for reinforced panels from fabric plies.

* - 0°	$D_{11}, N \cdot mm$	$D_{12}, N \cdot mm$	$D_{22}, N \cdot mm$	$D_{66}, N \cdot mm$	$N_{x.cr}, \frac{N}{mm}$
S31	64162	25789	63262	28483	289.11
S31+stringer	4458147	25789	63262	28483	289.11
* - 90°					
S31	63262	25789	64162	28483	289.11
S31+stringer	4457247	25789	64162	28483	289.11

5.5 SUMMARY PLOTS

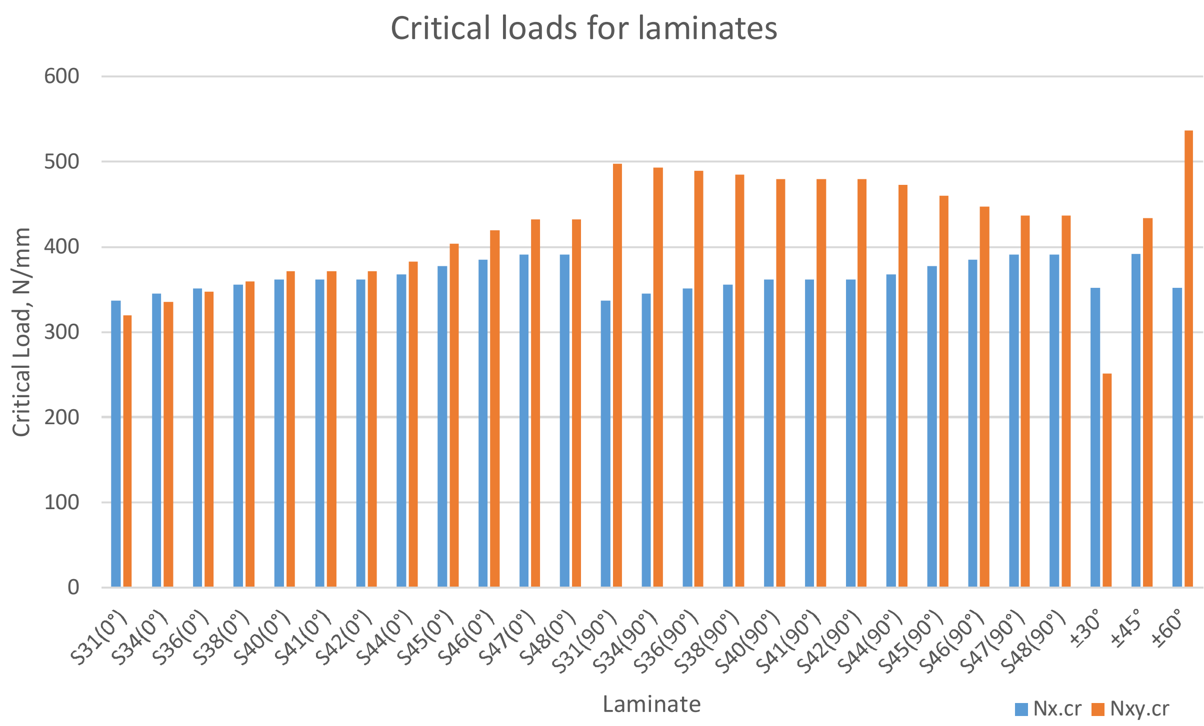


Fig. 36 Critical loads for all laminates from tape plies and winding laminates

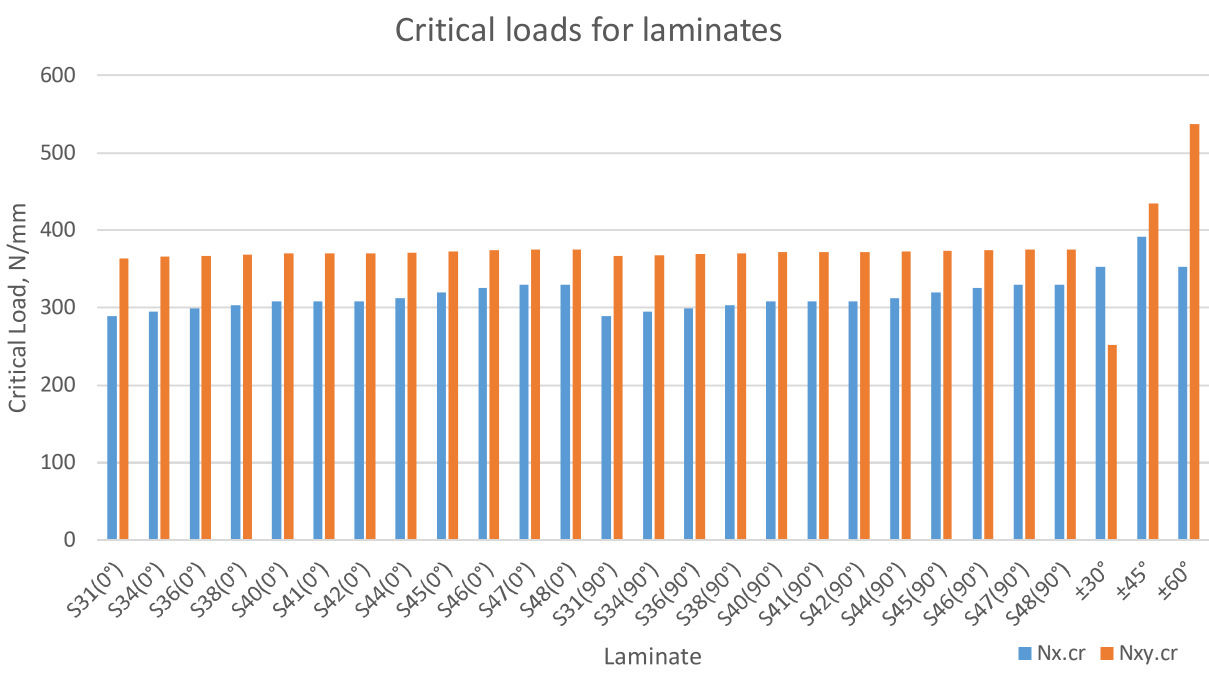


Fig. 37 Critical loads for all laminates from fabric plies and winding laminates

6 STRENGTH OF LAMINATE

There are many different failure criteria for composites, but the most useful is Mises-Hill-Tsai criteria. This is theory of distortion strain energy, which based on Mises criteria for isotropic materials, modified by Hill for anisotropic materials and applied for composites by Tsai. According to this theory, destruction in a monolayer will not occur if the stress acting in the material does not go beyond the surface defined as follows:

$$\left(\frac{\sigma_1}{F_1}\right)^2 + \left(\frac{\sigma_2}{F_2}\right)^2 + \left(\frac{\tau_{12}}{F^{su}}\right)^2 - \frac{\sigma_1 \cdot \sigma_2}{F_1^2} = 1 \tag{6.1}$$

where

$F_1 = F_1^{tu}$ when $\sigma_1 > 0,$ $\quad$ $F_2 = F_2^{tu}$ when $\sigma_2 > 0,$

$F_1 = F_1^{cu}$ when $\sigma_1 > 0,$ $\quad$ $F_2 = F_2^{cu}$ when $\sigma_2 > 0,$

where

F_1 and F_2 – allowable stresses for lamina in direction x_1 and x_2 respectively;

F^{tu} and F^{cu} – tension and compression ultimate stresses respectively;

F^{su} – shear ultimate stress;

σ_1 and σ_2 – actual normal stresses to limina in direction x_1 and x_2 respectively;

τ_{12} – actual shear stress to lamina.

This criterion gives a smooth piecewise limiting fracture surface formed from four generally different ellipsoids (see Fig.39 and 40), and takes into account the significant interaction between σ_1, σ_2 and τ_{12}. The considered criterion is used only to describe the behavior of composite with a weakly expressed anisotropy of strength properties.

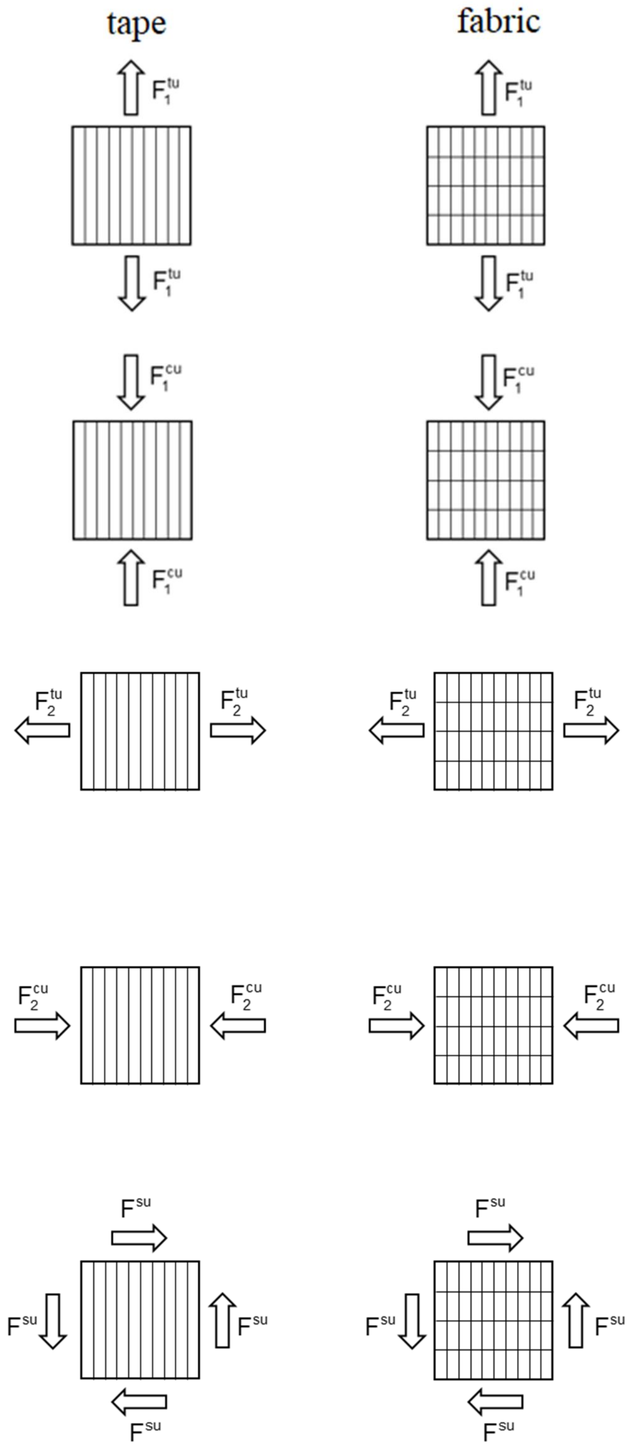


Fig. 38 Basic strength properties of lamina

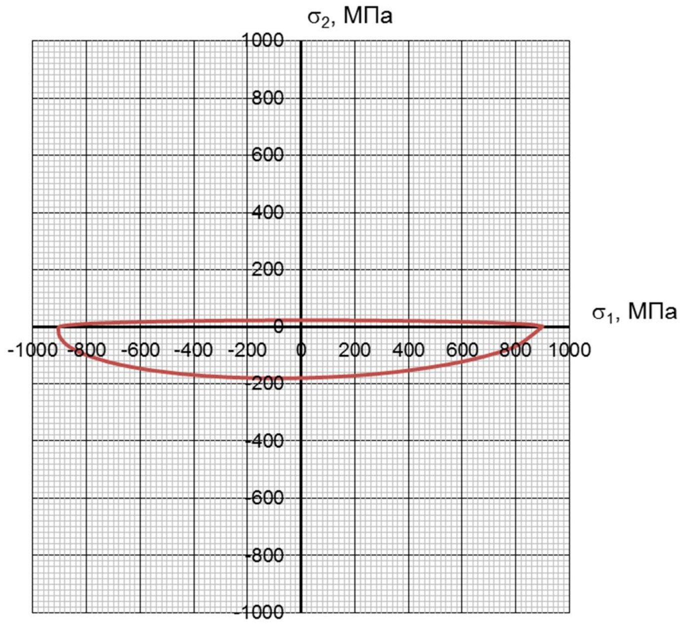


Fig. 39 Limiting contour of the fracture surface of the tape, corresponding to the Mises-Hill-Tsai criterion

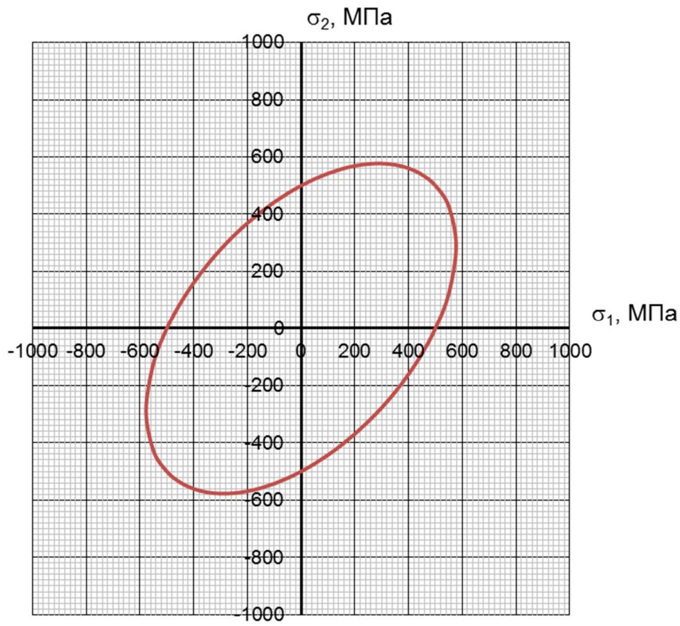


Fig. 40 Limiting contour of the fracture surface of the fabric, corresponding to the Mises-Hill-Tsai criterion

For example, Mises-Hill-Tsai criteria is used for plate with plies sequence S31.

Table 21 – Strength properties of tape

$F_1^{tu}\ MPa$	$F_1^{cu}\ MPa$	$F_2^{tu}\ MPa$	$F_2^{cu}\ MPa$	$F^{su}\ MPa$
900	900	22	180	70

Critical loads are used like applied loads for this panel:

$N_x = -337\frac{N}{mm}, \quad N_{xy} = 319\frac{N}{mm}.$

The deformations for laminate calculate by transforming Equ. 2.54

$$\begin{Bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{16} & \beta_{11} & \beta_{12} & \beta_{16} \\ \alpha_{12} & \alpha_{22} & \alpha_{26} & \beta_{21} & \beta_{22} & \beta_{26} \\ \alpha_{16} & \alpha_{26} & \alpha_{66} & \beta_{61} & \beta_{62} & \beta_{66} \\ \beta_{11} & \beta_{21} & \beta_{61} & \delta_{11} & \delta_{12} & \delta_{16} \\ \beta_{12} & \beta_{22} & \beta_{62} & \delta_{12} & \delta_{22} & \delta_{26} \\ \beta_{16} & \beta_{26} & \beta_{66} & \delta_{16} & \delta_{26} & \delta_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} \tag{6.2}$$

The [α], [β] and [δ] matrices are related to the [*A*], [*B*], [*D*] matrices by:

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{16} & \beta_{11} & \beta_{12} & \beta_{16} \\ \alpha_{12} & \alpha_{22} & \alpha_{26} & \beta_{21} & \beta_{22} & \beta_{26} \\ \alpha_{16} & \alpha_{26} & \alpha_{66} & \beta_{61} & \beta_{62} & \beta_{66} \\ \beta_{11} & \beta_{21} & \beta_{61} & \delta_{11} & \delta_{12} & \delta_{16} \\ \beta_{12} & \beta_{22} & \beta_{62} & \delta_{12} & \delta_{22} & \delta_{26} \\ \beta_{16} & \beta_{26} & \beta_{66} & \delta_{16} & \delta_{26} & \delta_{66} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix}^{-1} \tag{6.3}$$

For elements of [*A*], [*B*], [*D*] matrices see Table 32.

Since there are no moment applied to panel deformations k_x, k_y and k_{xy} are equal zero:

$k_x = 0, \quad k_y = 0, \quad k_{xy} = 0.$

Deformations due in-plane loads:

$\epsilon_x^0 = -0.003, \qquad \epsilon_y^0 = 0.0022, \qquad \gamma_{xy}^0 = -0.0042.$

Deformations of lamina calculates by following equation:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & s \cdot c \\ s^2 & c^2 & -s \cdot c \\ -2 \cdot s \cdot c & 2 \cdot s \cdot c & c^2 - s^2 \end{bmatrix} \cdot \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \tag{6.4}$$

Where

$c = \cos(\varphi)\,, s = \sin\,(\varphi)$

Stress of lamina calculated by Equ. 2.15:

$$\sigma_1 = C_{11}^0 \cdot \varepsilon_1 + C_{12}^0 \cdot \varepsilon_2$$

$$\sigma_2 = C_{12}^0 \cdot \varepsilon_1 + C_{22}^0 \cdot \varepsilon_2$$

$$\tau_{12} = C_{66}^0 \cdot \gamma_{12}$$

Table 22 – Calculation of margins by Mises-Hill-Tsai criteria

№	a	C_{11}^{0}	C_{12}^{0}	C_{22}^{0}	C_{66}^{0}	ε_1	ε_2	γ_{12}	σ_1	σ_2	τ_{12}	Margin
1	45	144097	3047	8464	5600	0,002	-0,003	0,005	237	-16	29	0.259
2	0	144097	3047	8464	5600	-0,003	0,002	0,004	-430	10	24	0.537
3	-45	144097	3047	8464	5600	-0,003	0,002	-0,005	-355	7	-29	0.430
4	0	144097	3047	8464	5600	-0,003	0,002	0,004	-430	10	24	0.537
5	-45	144097	3047	8464	5600	-0,003	0,002	-0,005	-355	7	-29	0.430
6	-45	144097	3047	8464	5600	-0,003	0,002	-0,005	-355	7	-29	0.430
7	45	144097	3047	8464	5600	0,002	-0,003	0,005	237	-16	29	0.259
8	45	144097	3047	8464	5600	0,002	-0,003	0,005	237	-16	29	0.259
9	-45	144097	3047	8464	5600	-0,003	0,002	-0,005	-355	7	-29	0.430
10	45	144097	3047	8464	5600	0,002	-0,003	0,005	237	-16	29	0.259
11	45	144097	3047	8464	5600	0,002	-0,003	0,005	237	-16	29	0.259
12	-45	144097	3047	8464	5600	-0,003	0,002	-0,005	-355	7	-29	0.430
13	45	144097	3047	8464	5600	0,002	-0,003	0,005	237	-16	29	0.259
14	45	144097	3047	8464	5600	0,002	-0,003	0,005	237	-16	29	0.259
15	-45	144097	3047	8464	5600	-0,003	0,002	-0,005	-355	7	-29	0.430
16	-45	144097	3047	8464	5600	-0,003	0,002	-0,005	-355	7	-29	0.430
17	0	144097	3047	8464	5600	-0,003	0,002	0,004	-430	10	24	0.537
18	-45	144097	3047	8464	5600	-0,003	0,002	-0,005	-355	7	-29	0.430
19	0	144097	3047	8464	5600	-0,003	0,002	0,004	-430	10	24	0.537
20	45	144097	3047	8464	5600	0,002	-0,003	0,005	237	-16	29	0.259

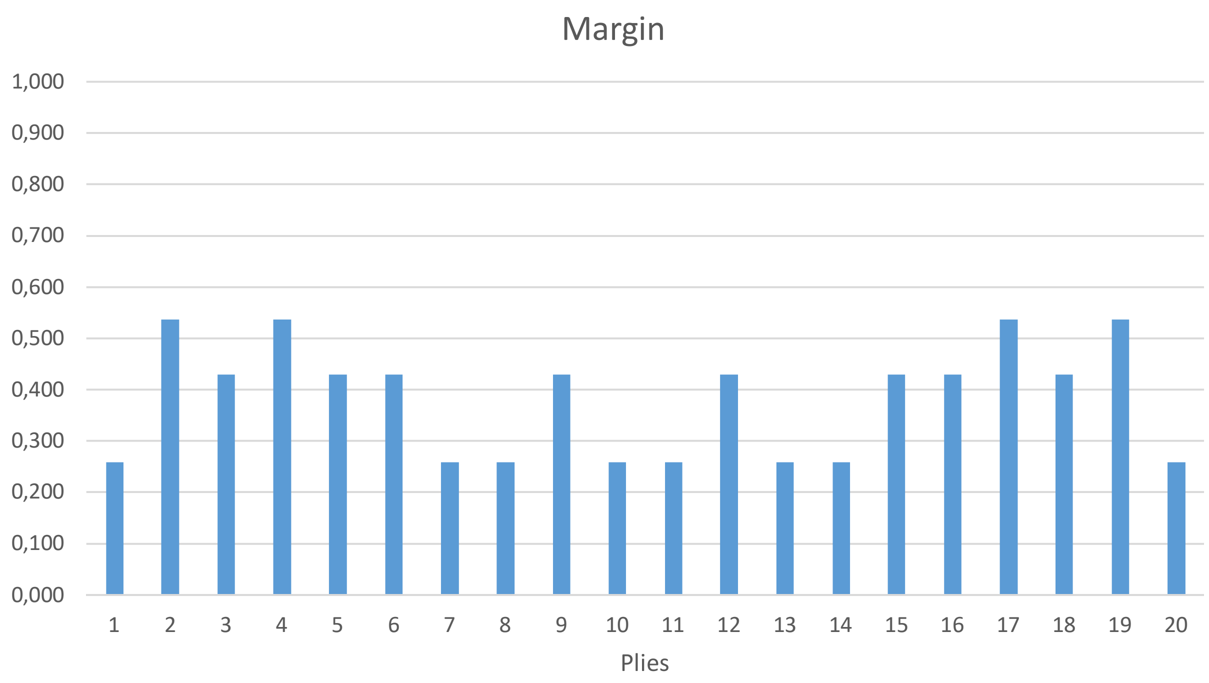


Fig. 41 Margins for all plies

7. DEVELOPMENT OF A STARTUP PROJECT

7.1 DESCRIPTION OF THE PROJECT IDEA

The section analyzes the marketing analysis of a startup project, identifies opportunities and feasibility of its introduction to the market.

Table 23 Description of the startup project

Project content	Areas of application	Benefits for users
Determination of the stiffness properties and critical loads of a composite panels	Mechanical engineering industry, Aircraft design	1) Accurate assessment 2) Speed and quality of results 3) Possibility to analyze composite panels with different sequences of plies and geometric properties.

The proposed technique allows determining the required level of loads for composite panels with different sequences of plies in a short time.

7.2. TECHNOLOGY AUDIT

It is possible to realize the idea of the project through field tests and statistical analysis. In the Table 24 the analysis of potential technical and economic advantages of this idea in comparison with the competitor # 1 (foreign colleagues in the field of aircraft and mechanical engineering industry).

Table 24 Determination of strong, weak and neutral characteristics of the project idea

№	Technical and economic characteristics of the idea	W	N	S
1	Cash	Competitor №1	-	My project
2	Method of assessment	-	Competitor №1	My project
3	Complexity of calculation	-	-	-

Table 25 Technological feasibility of the project idea

№	The idea of the project	Technology of its implementation	The presence of technology	Technology availability
1	Determination of the stiffness properties and critical load of composite panels	Simple interface	yes	yes
		Quick access in different devices		
The selected technology can be implemented.				

According to the indicators of the state of the market, we can conclude that this project is profitable.

7.3. ANALYSIS OF MARKET OPPORTUNITIES FOR LAUNCHING A STARTUP PROJECT

Determining the market opportunities that can be used in the market implementation of the project, and market threats that may impede the implementation of the project, is quite difficult, given that different methods of solving the task is an element of long-term scientific development of the industry. That is, to evaluate the potential market for a startup project is possible only in the long run, not based on clear numerical characteristics of the market. Let's analyze the market opportunities for the implementation of our project. To begin with, we will conduct a demand analysis: demand availability, volume and dynamics of market development Table 26.

Table 26 Preliminary description of a potential startup project market

№	Market state indicators	Characteristics
1	Number of main players, units	2
2	Total sales, UAH / unit	100
3	Market dynamics	increase
4	Sign-in restrictions	Absent
5	Specific requirements for standardization and certification	available
6	Average rate of return in the industry,%	100%

According to the indicators of the state of the market, we can conclude that this project is profitable.

Identification of potential customer groups

Potential customer groups can be roughly divided into primary and secondary customers. The primary group is the district and regional aircraft. In the future, we will identify potential customer groups per Table 27.

Table 27 Characteristics of potential clients of a startup project

№	The need that shapes the market	Target audience	Differences in behavior of different potential target customers	Consumer requirements for the product
1	Design of composite structure	Boeing subsidiaries	Finances	Speed of the determination and simplicity

Given the competitive situation, there is an opportunity to work in this market. To be competitive in the market, a project must have characteristics such as the speed of calculation and the availability of software.

Based on the analysis of competition conducted, and taking into account the characteristics of the idea of the project, consumer requirements for the table and factors of the marketing environment, determine and justify the list of factors of competitiveness. The analysis is formalized in Table 28.

Table 28 Rationale for competitiveness factors

№	Competitiveness factor	Rationale (citing factors that make the comparison of competing projects meaningful)
1	less need for costs	No need for repeat operations
2	Test accuracy	Improving results
3	The speed of calculation	Maximum resource depletion

According to the identified factors of competitiveness Table 28 we will analyze the strengths and weaknesses of my startup project Table 29.

The final stage of market analysis of project implementation opportunities is the compilation of SWOT analysis (Strength and Weak matrix, Troubles and Opportunities on the basis of selected market threats and opportunities, and strengths and weaknesses Table 29.

Table 29 Comparative analysis of strengths and weaknesses " Design of high-load single piece metal-composite compound of minimum mass”

№	Competitiveness factor	Points 1-20	Competitive rating of products compared to the project "Design of highly loaded metal-composite joints"						
			-3	-2	-1	0	1	2	3
1	less need for costs	20				●			
2	Accuracy of calculations	20			●				
3	Using the data obtained	20					●		
4	The accuracy of the calculation in the project	15					●		

The list of market threats and market opportunities is compiled on the basis of an analysis of threat factors and factors of the marketing environment. Market threats and market opportunities are the effects of factors and, by contrast, have not yet been realized in the market and are likely to occur.

Based on the SWOT analysis, market behavior alternatives are developed for launching a startup project to the market and an approximate optimal timing of their market implementation in view of potential competitors' projects that may be launched. The identified alternatives are analyzed in terms of timing and likelihood of receiving resources Table 30.

Table 30 Alternatives to market introduction of a startup project

№	An alternative to market behavior	The probability of receiving resources	Terms of implementation
1	Public review, review of existing studies (analogues), state approval	high	3 months
2	Publication, validation of the present experiment, state approval	high	10 month

From the above alternatives, we will choose the first one, because obtaining resources is simpler and more likely and the timing of implementation is shorter.

8. CONCLUSIONS

1. The excel template was developed to analyze deferent options of performance of carbon fiber composite panels.

2. The computing power of modern computers allows you to compare a vary large number options of performance of composite panels in short term of time. Therefore, special methods of selection optimal decision are not required.

3. Analysis showed that panels with high number of plies oriented by angle 45°, which located at a great distance from mid-plane, have higher compression critical loads and enough high shear critical loads.

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APPENDIX A

Input data

Table 31 – Elastic properties of tape lamina

Material	E_1, MPa	E_2, MPa	G_{12}, MPa	ν_{12}	δ_m, mm
Tape	143000	8400	5600	0.36	0.125

Table 32 – Calculation of stiffness properties of laminate S31 from tape plies

Ply	φ	E_1	E_2	G_{12}	ν_{12}	δ_m	ν_{21}	C_{11}^0	C_{12}^0	C_{22}^0	C_{66}^0	V_1	V_2	V_3	V_4
1	45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
2	0	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
3	-45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
4	0	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
5	-45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
6	-45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
7	45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
8	45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
9	-45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
10	45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
11	45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
12	-45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
13	45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
14	45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
15	-45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
16	-45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
17	0	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
18	-45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
19	0	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108
20	45	143000	8400	5600	0.36	0.125	0.02	144097	3047	8464	5600	60772	67816	15508	21108

Ply	φ	C_{11}	C_{12}	C_{16}	C_{22}	C_{26}	C_{66}	Z_{CL}	S_x	I_x	A_{11}	A_{12}	A_{16}	A_{22}	A_{26}	A_{66}
1	45	45264	34064	33908	45264	33908	36617	-1.1875	-0.148	0.176	5658	4258	4239	5658	4239	4577
2	0	144097	3047	0	8464	0	5600	-1.0625	-0.133	0.141	18012	381	0	1058	0	700
3	-45	45264	34064	-33908	45264	-33908	36617	-0.9375	-0.117	0.110	5658	4258	-4239	5658	-4239	4577
4	0	144097	3047	0	8464	0	5600	-0.8125	-0.102	0.083	18012	381	0	1058	0	700
5	-45	45264	34064	-33908	45264	-33908	36617	-0.6875	-0.086	0.059	5658	4258	-4239	5658	-4239	4577
6	-45	45264	34064	-33908	45264	-33908	36617	-0.5625	-0.070	0.040	5658	4258	-4239	5658	-4239	4577
7	45	45264	34064	33908	45264	33908	36617	-0.4375	-0.055	0.024	5658	4258	4239	5658	4239	4577
8	45	45264	34064	33908	45264	33908	36617	-0.3125	-0.039	0.012	5658	4258	4239	5658	4239	4577
9	-45	45264	34064	-33908	45264	-33908	36617	-0.1875	-0.023	0.005	5658	4258	-4239	5658	-4239	4577
10	45	45264	34064	33908	45264	33908	36617	-0.0625	-0.008	0.001	5658	4258	4239	5658	4239	4577
11	45	45264	34064	33908	45264	33908	36617	0.0625	0.008	0.001	5658	4258	4239	5658	4239	4577
12	-45	45264	34064	-33908	45264	-33908	36617	0.1875	0.023	0.005	5658	4258	-4239	5658	-4239	4577
13	45	45264	34064	33908	45264	33908	36617	0.3125	0.039	0.012	5658	4258	4239	5658	4239	4577
14	45	45264	34064	33908	45264	33908	36617	0.4375	0.055	0.024	5658	4258	4239	5658	4239	4577
15	-45	45264	34064	-33908	45264	-33908	36617	0.5625	0.070	0.040	5658	4258	-4239	5658	-4239	4577
16	-45	45264	34064	-33908	45264	-33908	36617	0.6875	0.086	0.059	5658	4258	-4239	5658	-4239	4577
17	0	144097	3047	0	8464	0	5600	0.8125	0.102	0.083	18012	381	0	1058	0	700
18	-45	45264	34064	-33908	45264	-33908	36617	0.9375	0.117	0.110	5658	4258	-4239	5658	-4239	4577
19	0	144097	3047	0	8464	0	5600	1.0625	0.133	0.141	18012	381	0	1058	0	700
20	45	45264	34064	33908	45264	33908	36617	1.1875	0.148	0.176	5658	4258	4239	5658	4239	4577

Ply #	φ	B_{11}	B_{12}	B_{16}	B_{22}	B_{26}	B_{66}	D_{11}	D_{12}	D_{16}	D_{22}	D_{26}	D_{66}
1	45	-6719	-5056	-5033	-6719	-5033	-5435	7986	6010	5982	7986	5982	6460
2	0	-19138	-405	0	-1124	0	-744	20357	430	0	1196	0	791
3	-45	-5304	-3992	3974	-5304	3974	-4291	4980	3748	-3731	4980	-3731	4029
4	0	-14635	-309	0	-860	0	-569	11914	252	0	700	0	463
5	-45	-3890	-2927	2914	-3890	2914	-3147	2682	2018	-2009	2682	-2009	2169
6	-45	-3183	-2395	2384	-3183	2384	-2575	1798	1353	-1347	1798	-1347	1454
7	45	-2475	-1863	-1854	-2475	-1854	-2002	1090	821	817	1090	817	882
8	45	-1768	-1331	-1325	-1768	-1325	-1430	560	421	419	560	419	453
9	-45	-1061	-798	795	-1061	795	-858	206	155	-155	206	-155	167
10	45	-354	-266	-265	-354	-265	-286	29	22	22	29	22	24
11	45	354	266	265	354	265	286	29	22	22	29	22	24
12	-45	1061	798	-795	1061	-795	858	206	155	-155	206	-155	167
13	45	1768	1331	1325	1768	1325	1430	560	421	419	560	419	453
14	45	2475	1863	1854	2475	1854	2002	1090	821	817	1090	817	882
15	-45	3183	2395	-2384	3183	-2384	2575	1798	1353	-1347	1798	-1347	1454
16	-45	3890	2927	-2914	3890	-2914	3147	2682	2018	-2009	2682	-2009	2169
17	0	14635	309	0	860	0	569	11914	252	0	700	0	463
18	-45	5304	3992	-3974	5304	-3974	4291	4980	3748	-3731	4980	-3731	4029
19	0	19138	405	0	1124	0	744	20357	430	0	1196	0	791
20	45	6719	5056	5033	6719	5033	5435	7986	6010	5982	7986	5982	6460

Components of stiffness matrix of laminate calculates by summing of components for each ply:

$$A_{ij} = \sum_{k=1}^{n} (A_{ij})_k \qquad B_{ij} = \sum_{k=1}^{n} (B_{ij})_k \qquad D_{ij} = \sum_{k=1}^{n} (D_{ij})_k$$

where

k – k-th ply; n – number of plies.

$$\begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} = \begin{bmatrix} 162576 & 69652 & 0 & 0 & 0 & 0 \\ 69652 & 94760 & 0 & 0 & 0 & 0 \\ 0 & 0 & 76034 & 0 & 0 & 0 \\ 0 & 0 & 0 & 103206 & 30461 & 0 \\ 0 & 0 & 0 & 30461 & 42454 & 0 \\ 0 & 0 & 0 & 0 & 0 & 33785 \end{bmatrix}$$

Components of stiffness matrix for other angels obtained also by same way.

Table 33 – Stiffness properties of laminate S31 from tapes, depending on $\varphi°$

φ	D_{11}	D_{22}	D_{66}
0	103206	42454	33785
5	103125	43296	33405
10	102848	45760	32311
15	102287	49674	30635
20	101306	54767	28579
25	99750	60699	26391
30	97469	67093	24335
35	94346	73568	22659
40	90326	79776	21565
45	85431	85431	21185
50	79776	90326	21565
55	73568	94346	22659
60	67093	97469	24335
65	60699	99750	26391
70	54767	101306	28579
75	49674	102287	30635
80	45760	102848	32311
85	43296	103125	33405
90	42454	103206	33785

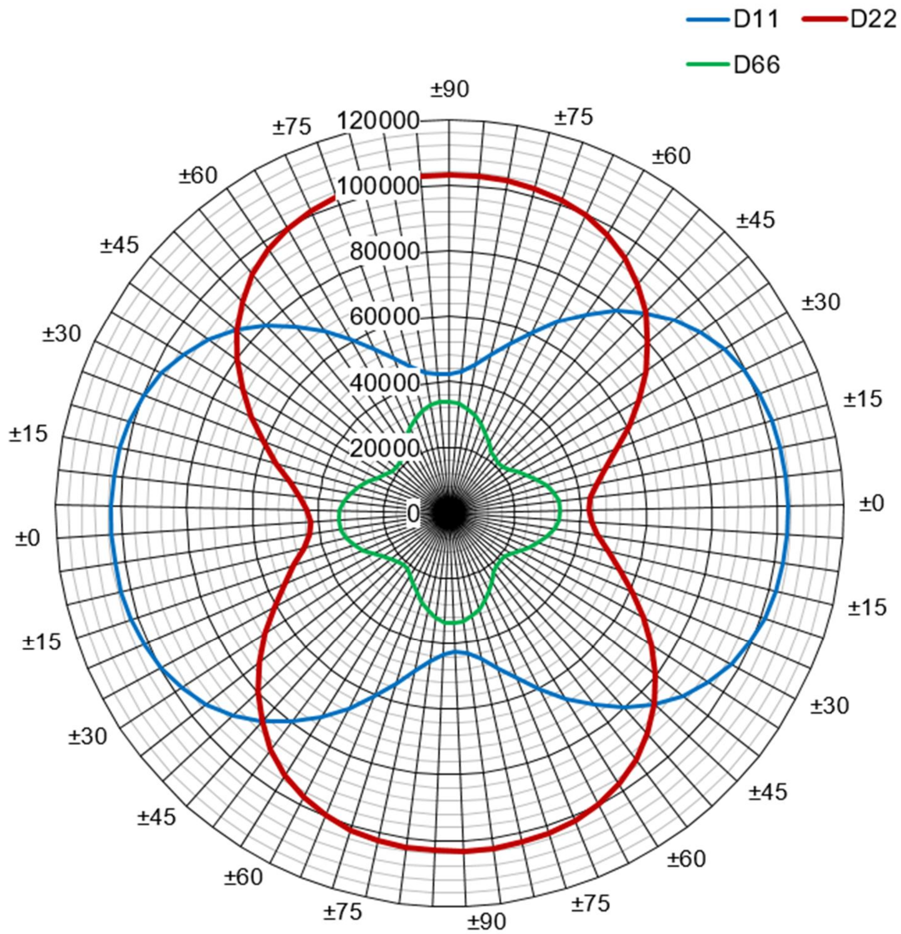

Fig. 42 Stiffness properties of laminate S31 from tapes, depending on φ° in polar coordinate system